Total absorption peak by use of a rigid frame porous layer backed by a rigid multi-irregularities grating.

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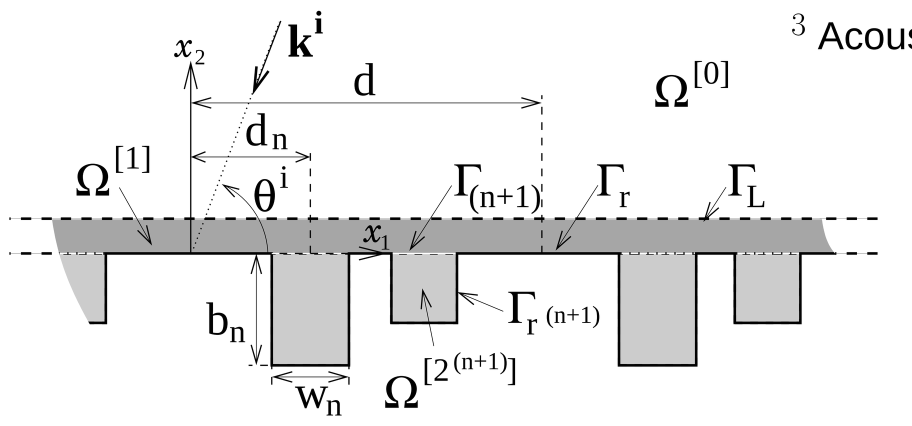


1 Introduction

This work was initially motivated by a design problem connected to the determination of the optimal profile of discontinuous spatial distribution of porous materials and geometric properties for the absorption of sound.
Here, we investigate theoretically, numerically and experimentally the influence on the absorption coefficient of a multi-component gratings against which a rigid frame porous layer is glued.

2 The mathematical problem

The plane wave nature of the incident wave and the periodic nature of $\cup_{n\in\mathcal{N}}\Omega^{[2^{(n)}]}$ imply the **Floquet relation**

$$p(x_1+qd,x_2)=p(x_1,x_2)e^{\mathrm{i}k_1^i qd}\,;\ \forall\mathbf{x}\in\mathbb{R}^2\,;\ \forall q\in\mathbb{Z}\,.$$

Rigid frame porous materials M are modeled using the **Johnson-Champoux-Allard model**.

2.1 Evaluation of the fields

Separation of variables, the radiation conditions, and the Floquet theorem lead to the representations:

$$p^{[0]}=\sum_{q\in\mathbb{Z}}\left[e^{-\mathrm{i}k_{2q}^{[0]}(x_2-L)}\delta_q+R_q e^{\mathrm{i}k_{2q}^{[0]}(x_2-L)}\right]e^{\mathrm{i}k_{1q}x_1}\,,$$

$$p^{[1]}=\sum_{q\in\mathbb{Z}}\left[f_p e^{-\mathrm{i}k_{2q}^{[1]}x_2}+g_p e^{\mathrm{i}k_{2q}^{[1]}x_2}\right]e^{\mathrm{i}k_{1q}x_1}\,,$$

wherein δ_q is the Kronecker symbol and $k_{1q}=k_1^i+2q\pi/d$.

The pressure fields $p^{[2^{(n)}]}$, admits the pseudo-modal representation, that already accounts for the boundary conditions on $\Gamma_{r^{(n)}}$:

$$p^{[2^{(n)}]}=\sum_{m=0}^{\infty}B_m^{(n)}\cos\left(k_{1m}^{[2^{(n)}]}(x_1-d_n+w_n/2)\right)\cos\left(k_{2m}^{[2^{(n)}]}(x_2+b_n)\right)\,,$$

wherein $k_{1m}^{[2^{(n)}]}=m\pi/w_n$.

Applying the continuity of the pressure field and of the normal component of the velocity across Γ_L and Γ_0, give rise to the linear set of equations, which may be written in the matrix form, when denoting by $\mathbf{B}$ the infinite column matrix of components $B_m^{(n)}$

$$(\mathbf{A}-\mathbf{C})\,\mathbf{B}=\mathbf{F}\,. \qquad (1)$$

Once (1) is solved for $B_m^{(n)}$, R_q, f_q and g_q in terms of $B_m^{(n)}$ can be evaluated and in particular

$$R_q=\delta_q\frac{\alpha_q^{[0]}\cos\left(k_{2q}^{[1]}L\right)+\mathrm{i}\alpha_q^{[1]}\sin\left(k_{2q}^{[1]}L\right)}{D_q}+\sum_{n\in\mathcal{N}}\sum_{m=0}^{\infty}\frac{\mathrm{i}w_n\alpha_m^{[2^{(n)}]}}{dD_q}B_m^{(n)}\sin\left(k_{2m}^{[2^{(n)}]}b_n\right)I_{qm}^{-(n)}e^{-\mathrm{i}k_{1q}(d_n-w_n/2)}\,,$$

wherein $D_q=\alpha_q^{[0]}\cos\left(k_{2q}^{[1]}L\right)-\mathrm{i}\alpha_q^{[1]}\sin\left(k_{2q}^{[1]}L\right)$.

2.2 Acoustic properties

In case of an incident plane wave with spectrum A^i, and irregularities filled with the air medium the conservation of energy relation takes the form

$$1=\mathcal{A}+\mathcal{R}\,,$$

with $\mathcal{R}$ and $\mathcal{A}$ the hemispherical reflection and absorption coefficients respectively defined by

$$\mathcal{R}=\sum_{q\in\mathbb{Z}}\frac{\mathrm{Re}\left(k_{2q}^{[0]}\right)|R_q|^2}{k_2^{[0]i}\ |A^i|^2}=\sum_{q=-\tilde{q}_-}^{\tilde{q}_+}\frac{k_{2q}^{[0]}\ |R_q|^2}{k_2^{[0]i}\ |A^i|^2}\,,$$

where $\tilde{q}_\mp$ are such that $\tilde{q}_\mp<d/2\pi\left(k^{[0]}\pm k_1^i\right)<\tilde{q}_\mp+1$ and $\mathcal{A}=\mathcal{A}_D+\mathcal{A}_S$ where $\mathcal{A}_D$ corresponds to the **inner absorption of the domain** $\Omega^{[1]}$ and $\mathcal{A}_S$ accounts for the **absorption induced by the viscous dissipation at the interfaces** Γ_L and $\Gamma_{(n)}$. Because of the complicated shape of $\Omega^{[1]}$ and $\Omega^{[2^{(n)}]}$, but also of the non-vanishing term $\mathcal{A}_S$, $\mathcal{A}$ will be calculated by $\mathcal{A}=1-\mathcal{R}$.

3 Modal analysis

The **modes of the configuration without irregularities** of the rigid backing, whose dispersion relation is

$$D^i=\alpha^{[0]i}\cos\left(k_2^{[1]i}L\right)-\mathrm{i}\alpha^{[1]i}\sin\left(k_2^{[1]i}L\right)=0\,,$$

cannot be excited by a plane incident wave initially traveling in the air medium. **In the diffusion regime**, i.e. for frequencies below the Biot Frequency, **any mode exists**.
When the rigid backing presents one irregularity per spatial period and when a correct representation of the field can be given by accounting only for the fundamental pseudo-mode of the irregularity, the dispersion relation $\det(\mathbf{A}-\mathbf{C})=0$ reduces to

$$1-\sum_{q\in\mathbb{Z}}\frac{\mathrm{i}w}{d}\alpha^{[2]}\tan(k^{[2]}b)\mathrm{sinc}^2\left(k_{1q}\frac{w}{2}\right)\times\left[\alpha_q^{[1]}\frac{\alpha_q^{[0]}\cos\left(k_{2q}^{[1]}L\right)-\mathrm{i}\alpha_q^{[1]}\sin\left(k_{2q}^{[1]}L\right)}{\alpha_q^{[1]}\cos\left(k_{2q}^{[1]}L\right)-\mathrm{i}\alpha_q^{[0]}\sin\left(k_{2q}^{[1]}L\right)}\right]^{-1}=0\,,\ \text{wherein}\ \alpha_0^{[2^{(n)}]}=k^{[2]}/\rho^{[2]}=\alpha^{[2]}\,. \qquad (2)$$

By referring to **Cutler mode**, Eq. (2) is satisfied (in the non-dissipative case) when its denominator is purely imaginary and vanishes. These conditions are achieved when $|k_{1q}|\in[k^{[0]},\mathrm{Re}\left(k^{[1]}\right)]$ and when either $D_q=0$ or $\alpha_q^{[1]}=0$, which respectively corresponds to **modified modes of the backed-layer (MMBL)** and to **modes of the grating (MG)**.
In addition, the mode of the global configuration can be understood either as a modified **mode of the irregularities (MI)** satisfying $\cos(k_{2m}^{[2^{(n)}]}b)=0$, or as a MMBL, satisfying $D_q=0$.

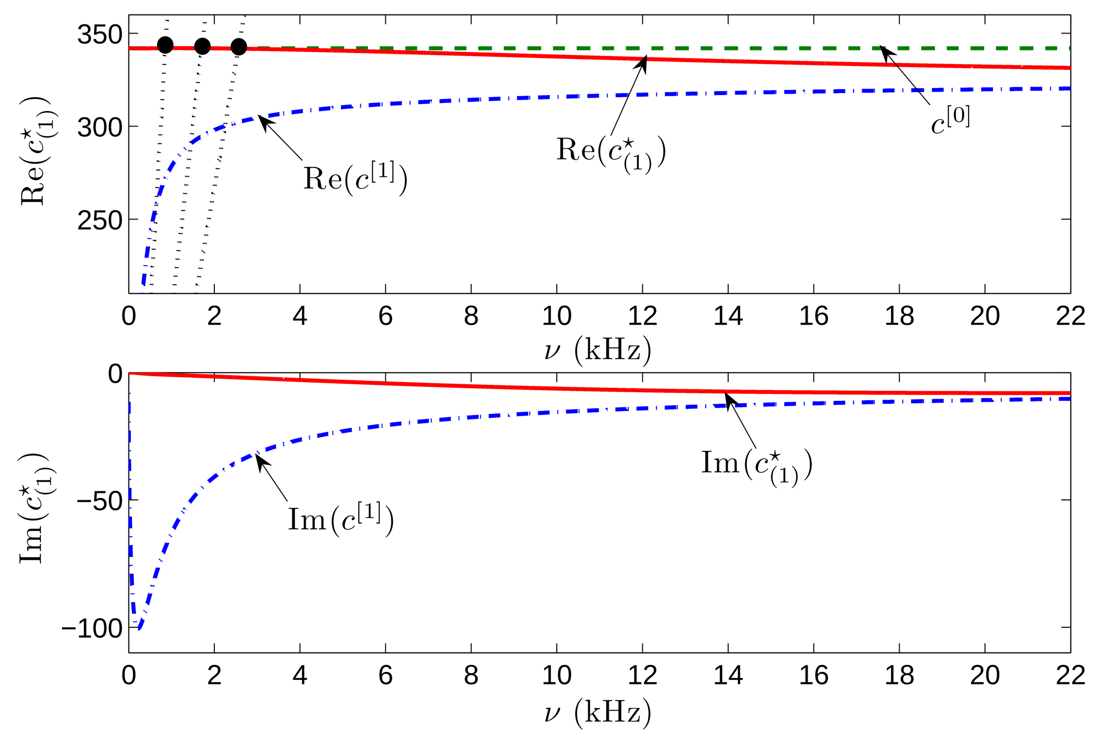


Figure 1: Real and imaginary part of the root of the dispersion relation in absence of irregularities $c^\star_{(1)}$. Real part of the first three modified modes of the layer $c^\star_{(1,q)}$, $q=1,...,3$, for $d=40$ cm are pointed out by dot.

4 Numerical results, experimental validation and discussion

The **spatial period is** $\mathbf{d=40}$ **cm**. The irregularities are filled with air, i.e. the ambient ($M^{[0]}$ and $M^{[2]}$) and saturating fluid is air. A $\mathbf{L=8}$ **mm thick low resistivity foam layer**, whose parameters are reported in Table 1, was used.
The MMBL will be around $\nu_{(1,1)}\approx 850$ Hz, $\nu_{(1,2)}\approx 1700$ Hz, $\nu_{(1,3)}\approx 2550$ Hz,...), while the MG should be excited around $\nu_1\approx 700$ Hz, $\nu_2\approx 1500$ Hz and $\nu_3\approx 2300$ Hz.

	ϕ	α_∞	Λ (μm)	Λ' (μm)	σ (Nsm^{-4})	$f_c=\omega_c/2\pi$ (Hz)
P1	0.96	1.07	273	672	2843	334

Table 1: Acoustical parameters of the porous material.

4.1 One irregularity per d

When the **fundamental MI stands at** $\nu_{(1,2)}$, i.e. $c^{[2]}/4b_1\approx\nu_{(1,2)}$ from which we can determine b_1, and the **second MI stands at** $\nu_{(1,3)}$, i.e. $\frac{c^{[2]}}{2\pi}\sqrt{\left(\frac{\pi}{2b_1}\right)^2+\left(\frac{\pi}{w_1}\right)^2}\approx\nu_{(1,3)}$ from which we can determine w_1, the **absorption at the frequency of the first MMBL is close to one**, Fig. 2.

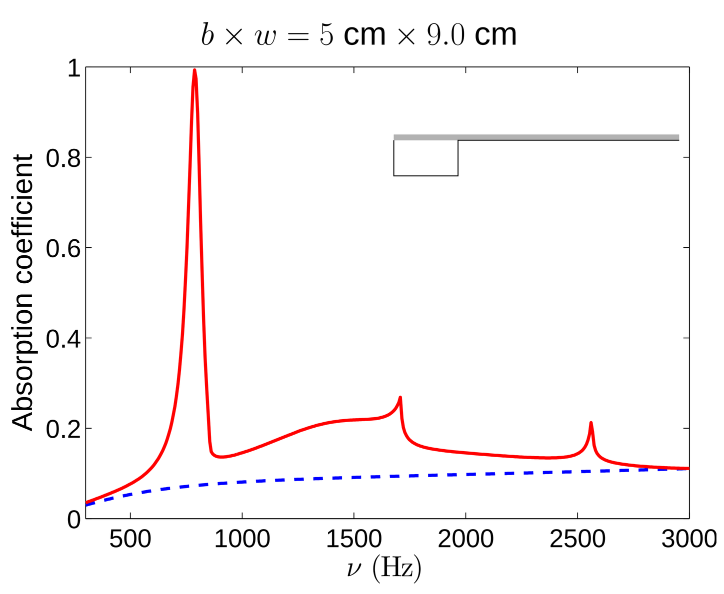
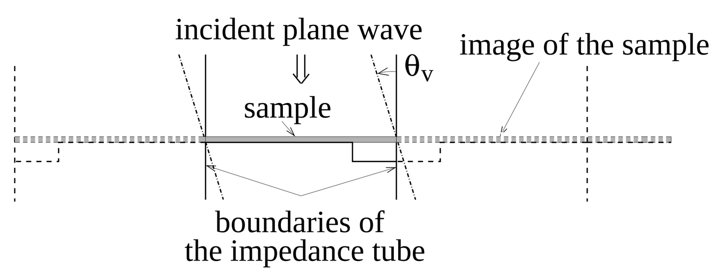


Figure 2: Absorption coefficient of the foam layer (– – –) backed with a rigid flat plate and (—) backed with a rigid grating.

4.2 Experimental validation

Here, experimental validation were carried out by use of an **impedance tube with a square cross section**, 20 cm $\times$ 20 cm, whose cut-off frequency is $850Hz$. The latter corresponds to a wavelength of 40 cm.



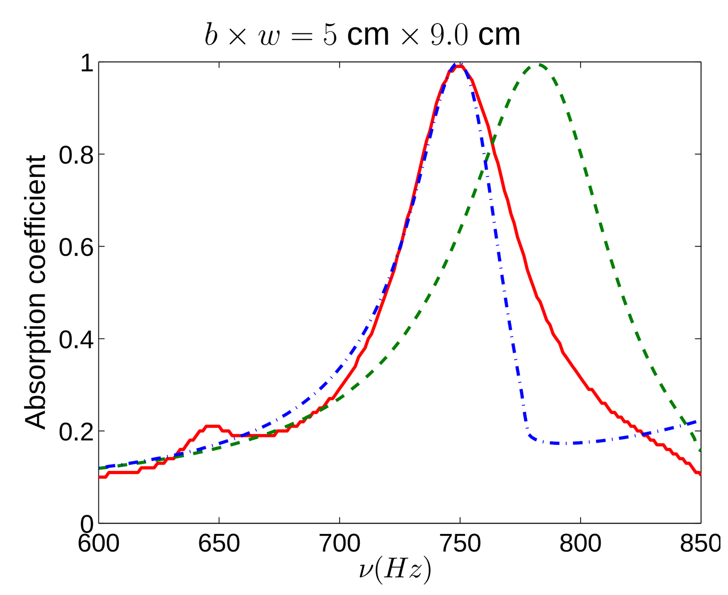


Figure 3: Absorption coefficients as measured experimentally (—), as calculated for $\theta_v=0^o$ (– – –) and as calculated for $\theta_v=5^o$ (– · –).

4.3 Two or more irregularities per d

Depending on the center-to-center distance, the addition of

- a **second irregularity of a larger high** leads to **another total absorption peak** at the fundamental MI, Fig. 4 a).
- a **second irregularity of smaller size**, mainly leads to **higher peak either of high order MMBL or of MI**, Fig. 4 b).
- **periodically spaced identical irregularities of smaller size** lead to a **higher frequency peak of absorption** associated with the excitation of the corresponding quasi-MMBL, Fig. 4 c).

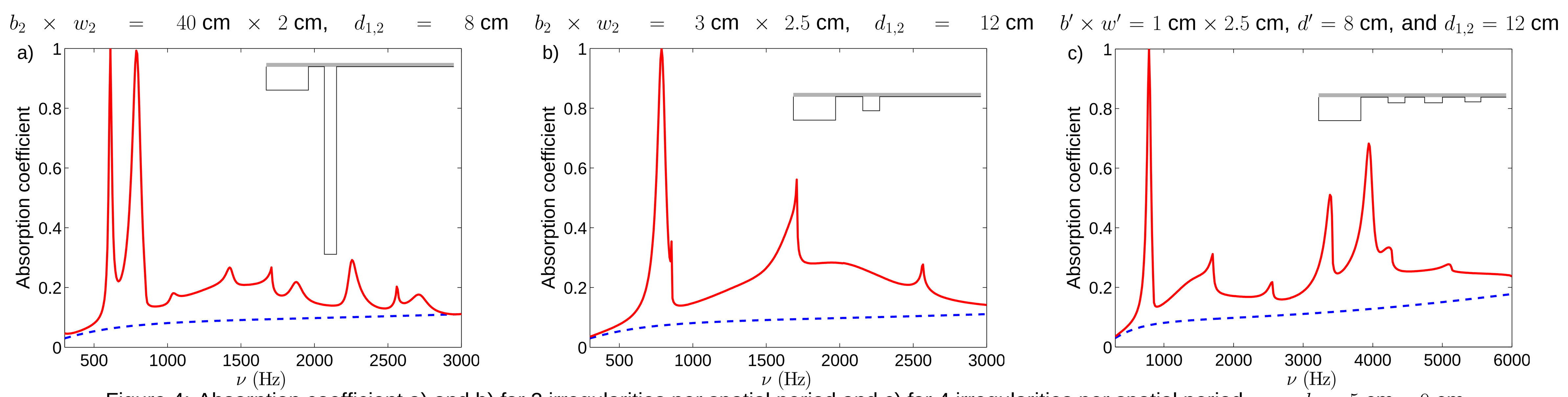


Figure 4: Absorption coefficient a) and b) for 2 irregularities per spatial period and c) for 4 irregularities per spatial period. $w_1\times b_1=5$ cm $\times$ 9 cm.

5 Conclusion

We show, especially through a modal analysis carried out in the case of only one irregularity per spatial period, that the gratings lead to excitation of modes, whose frequency depends both on the characteristic of the surrounding medium and of the characteristics of the porous layer and on the spatial period of the configuration d. These modes, whose structures are close to the one of the modes of the layer, can lead to a total absorption peak.