Radiation Trapping in a cold atomic gas at finite temperature

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1 Introduction

We are interesting in the **trapping of photons** in a scattering, resonant and dynamic atomic vapour [1]:

- **Goal of this work:** understanding the experiences of G. Labeyrie [2]
- **Main assumptions:**
  - Atoms treated as classical ponctual scatterers;
  - Doppler effect taken into account, recoil neglected;
  - Magneto-Optical Trap (MOT) regime considered, dilute system.

  ⇒ **Velocity of the scatterers:** $\boxed{v_{\text{recoil}} \ll v \ll \lambda_0\Gamma}$ where Γ is the spontaneous decay rate and λ_0 the wavelength.
- This induces a **partial frequency redistribution** (PFR) at each scattering event ⇒ Behavior very far from the complete frequency redistribution (CFR) case [3].

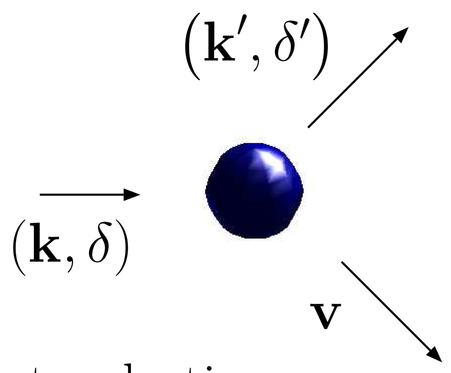
2 Transport equation derivation

- We start from **first principles** generalized to the case of moving atoms.
- **Generalized scattering operator** for a two-level atom of speed $\mathbf{v}$:

$$t_m^{\mathbf{v}}\left(\mathbf{k},\mathbf{k}',\delta,\delta'\right) = 2\pi t\left(\mathbf{k},\mathbf{k}',\delta-\mathbf{k}\cdot\mathbf{v}\right)\boldsymbol{\delta}\left[\delta'-\delta-\left(\mathbf{k}'-\mathbf{k}\right)\cdot\mathbf{v}\right]$$

where the scattering operator is given by the polarisability

$$t\left(\mathbf{k},\mathbf{k}',\delta\right) = -\frac{\omega^2}{c_0^2}\alpha\left(\delta\right) \sim \frac{4\pi}{k_0}\frac{\Gamma/2}{\delta+i\Gamma/2}$$



with $k_0 = 2\pi/\lambda_0$ and $\delta = \omega-\omega_0$, ω_0 being the resonant pulsation.

- **Generalized Bethe-Salpeter equation:**

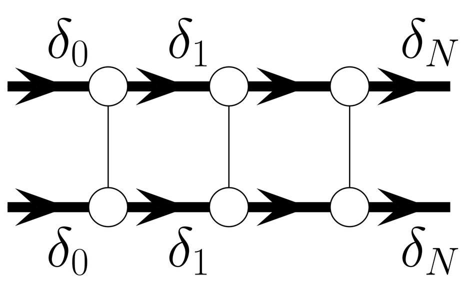
$$\left\langle E\left(\mathbf{r}_1,t_1\right)E^*\left(\mathbf{r}_2,t_2\right)\right\rangle = \int\cdots\int\left\langle G\left(\mathbf{r}_1,\mathbf{r}_1',t_1,t_1'\right)\right\rangle\left\langle G^*\left(\mathbf{r}_2,\mathbf{r}_2',t_2,t_2'\right)\right\rangle \times K_m\left(\mathbf{r}_1',\boldsymbol{\rho}_1,\mathbf{r}_2',\boldsymbol{\rho}_2,t_1',\tau_1,t_2',\tau_2\right)\left\langle E\left(\boldsymbol{\rho}_1,\tau_1\right)E^*\left(\boldsymbol{\rho}_2,\tau_2\right)\right\rangle \times \mathrm{d}^3\mathbf{r}_1'\mathrm{d}^3\mathbf{r}_2'\mathrm{d}^3\boldsymbol{\rho}_1\mathrm{d}^3\boldsymbol{\rho}_2\,\mathrm{d}t_1'\mathrm{d}t_2'\mathrm{d}\tau_1\mathrm{d}\tau_2$$

where E is the electric field and K_m is the generalized vertex intensity.

- **Transport equation for the diffuson:**

$$\boxed{\left[\frac{1}{c_0}\frac{\partial}{\partial t}+\mathbf{u}\cdot\boldsymbol{\nabla}_{\mathbf{r}}\right]I\left(\mathbf{u},\mathbf{r},\delta,t\right) = -\int_{t'=0}^{\infty}\int_{\mathbf{v}}\mu_e\left(\delta-k_0\mathbf{u}\cdot\mathbf{v},t'\right)g\left(\mathbf{v}\right)\times I\left(\mathbf{u},\mathbf{r},\delta,t-t'\right)\mathrm{d}\mathbf{v}\mathrm{d}t' + \int_{t'=0}^{\infty}\int_{4\pi}\int_{\mathbf{v}}\mu_s\left(\delta-k_0\mathbf{u}\cdot\mathbf{v},t'\right)g\left(\mathbf{v}\right)\times I\left(\mathbf{u}',\mathbf{r},\delta+k_0\left(\mathbf{u}'-\mathbf{u}\right)\cdot\mathbf{v},t-t'\right)\mathrm{d}\mathbf{v}\mathrm{d}\mathbf{u}'\mathrm{d}t'.}$$

$$\begin{cases}\mu_e\left(\delta,\Omega\right) = \dfrac{i\rho}{2k_0}\left\{t\left(\delta+\dfrac{\Omega}{2}\right)-t^*\left(\delta-\dfrac{\Omega}{2}\right)\right\},\\[2mm] \mu_s\left(\delta,\Omega\right) = \dfrac{\rho}{4\pi}t\left(\delta+\dfrac{\Omega}{2}\right)t^*\left(\delta-\dfrac{\Omega}{2}\right)\end{cases}$$

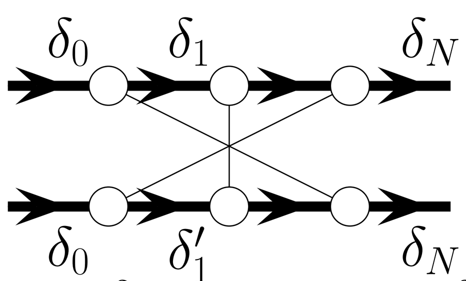


and $g\left(\mathbf{v}\right)$ is the velocity distribution of standard deviation $\bar{v}$. I is the Fourier transform of the correlation function of the field.

- **Transport equation for the cooperon:**

$$\boxed{\left[i\frac{\delta'-\delta}{c_0}+\mathbf{u}\cdot\boldsymbol{\nabla}_{\mathbf{r}}\right]I\left(\mathbf{u},\mathbf{r},\delta,\delta'\right) = -\int_{\mathbf{v}}\mu_e\left(\delta-k_0\mathbf{u}\cdot\mathbf{v},\delta'+k_0\mathbf{u}\cdot\mathbf{v}\right)g\left(\mathbf{v}\right)\times I\left(\mathbf{u},\mathbf{r},\delta,\delta'\right)\mathrm{d}\mathbf{v} + \int_{4\pi}\int_{\mathbf{v}}\mu_s\left(\delta-k_0\mathbf{u}\cdot\mathbf{v},\delta'+k_0\mathbf{u}\cdot\mathbf{v}\right)g\left(\mathbf{v}\right)\times I\left(\mathbf{u}',\mathbf{r},\delta+k_0\left(\mathbf{u}'-\mathbf{u}\right)\cdot\mathbf{v},\delta'-k_0\left(\mathbf{u}'-\mathbf{u}\right)\cdot\mathbf{v}\right)\mathrm{d}\mathbf{v}\mathrm{d}\mathbf{u}'\mathrm{d}t'.}$$

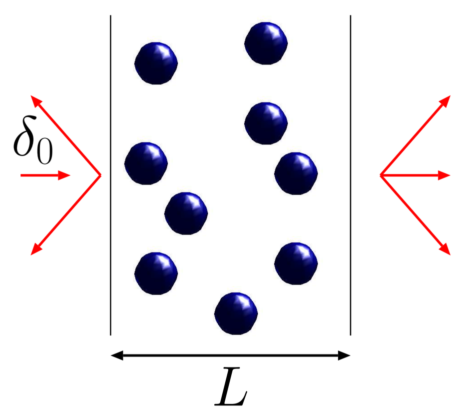
$$\begin{cases}\mu_e\left(\delta,\delta'\right) = \dfrac{i\rho}{2k_0}\left\{t\left(\delta\right)-t^*\left(\delta'\right)\right\},\\[2mm] \mu_s\left(\delta,\delta'\right) = \dfrac{\rho}{4\pi}t\left(\delta\right)t^*\left(\delta'\right)\end{cases}$$



- **Scattering mean-free path**: $\ell\left(\delta\right) = k_0^2/\left(4\pi\rho\right)\left[1+4\delta^2/\Gamma^2\right]$, $\ell_0 = \ell\left(0\right)$.

3 Monte Carlo simulations

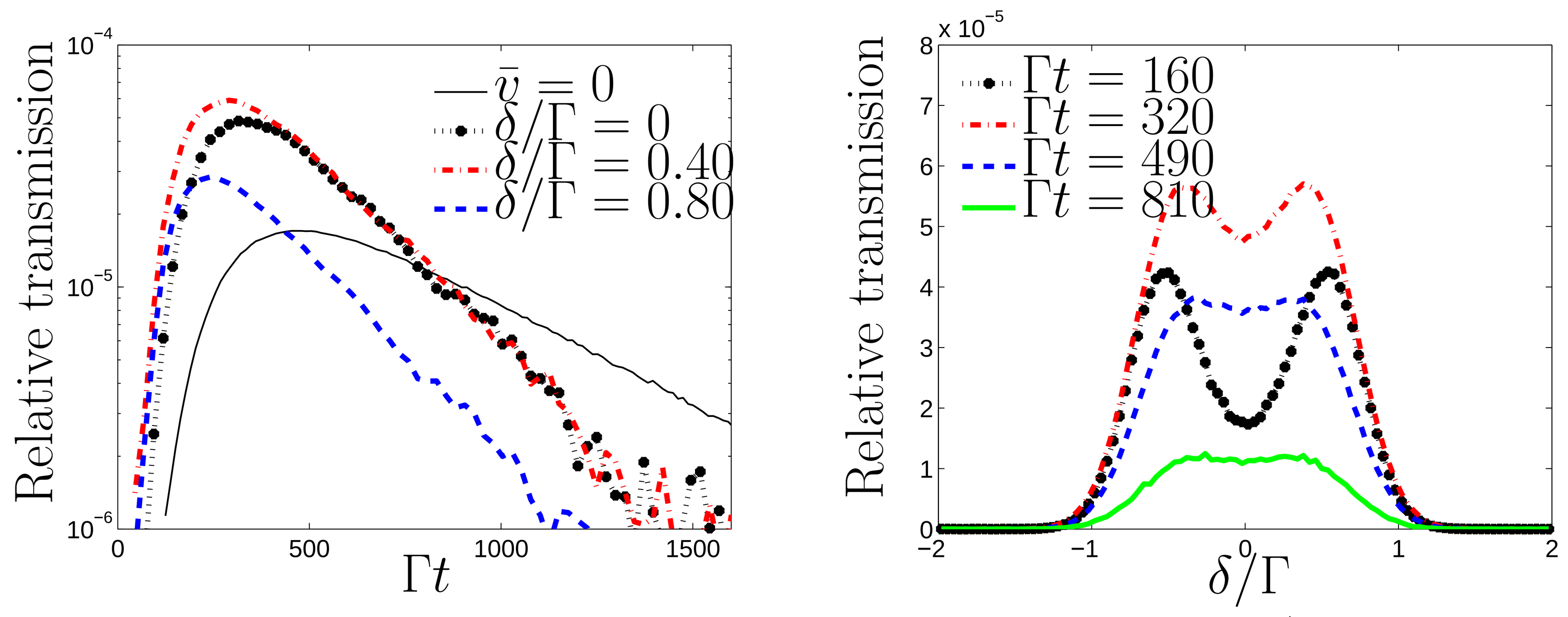
- **System properties:**
  - Slab geometry of thickness L;
  - Incident pulse at δ_0;
  - Parameters corresponding to Rubidium: $\omega_0 = 2.42\times10^{15}\,\text{rad.s}^{-1}$, $\Gamma = 3.7\times10^7\,\text{s}^{-1}$.



- **Numerical method for the diffuson:**

Temporal Fourier transform → Monte Carlo simulation → Inverse temporal Fourier transform

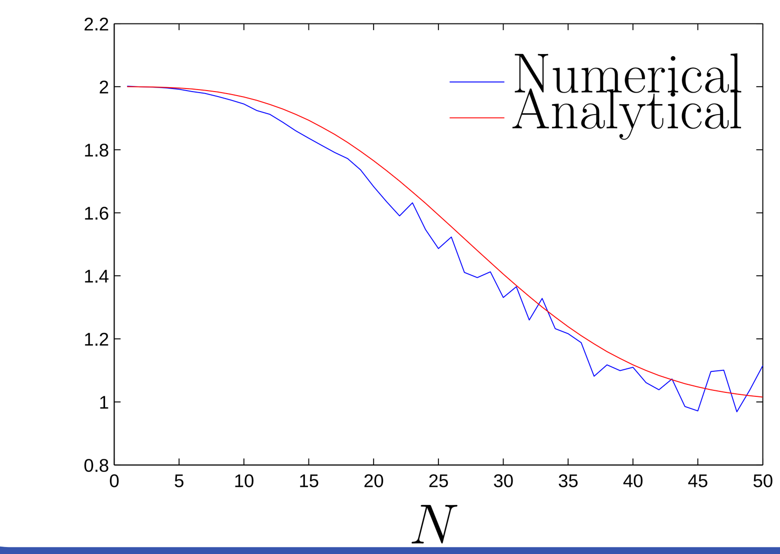
Transmitted flux for $L/\ell_0 = 40$ and $k_0\bar{v}/\Gamma = 0.02$



  - **Long time behavior:** exponential decay ($\tau = 262\Gamma^{-1}$). Competition between L, $\bar{v}$ and Γ ⇒ Key parameter: $\boxed{\mathcal{A} = k_0\bar{v}L/\left[\Gamma\ell_0\right]}$
    - ∗ If $\mathcal{A} \ll 1$ ⇒ quadratic regime ⇒ $\tau \propto L^3$;
    - ∗ If $\mathcal{A} \gg 1$ ⇒ Doppler regime.
  - **Spectral behavior:** competition between L, $\bar{v}$ and Γ.
    - ∗ For large L, two peaks are present. The trap of photons is more efficient at resonance.
    - ∗ For small L, only one peak is present.
- **Numerical method for the cooperon:**

Monte Carlo simulation for the diffuson → Results used to compute the cooperon

CBS contrast versus scattering order N for $k_0\bar{v}/\Gamma = 0.01$



For: $k_0\bar{v}\sqrt{N}/\Gamma \ll 1$, the constrast of the cone decreases like

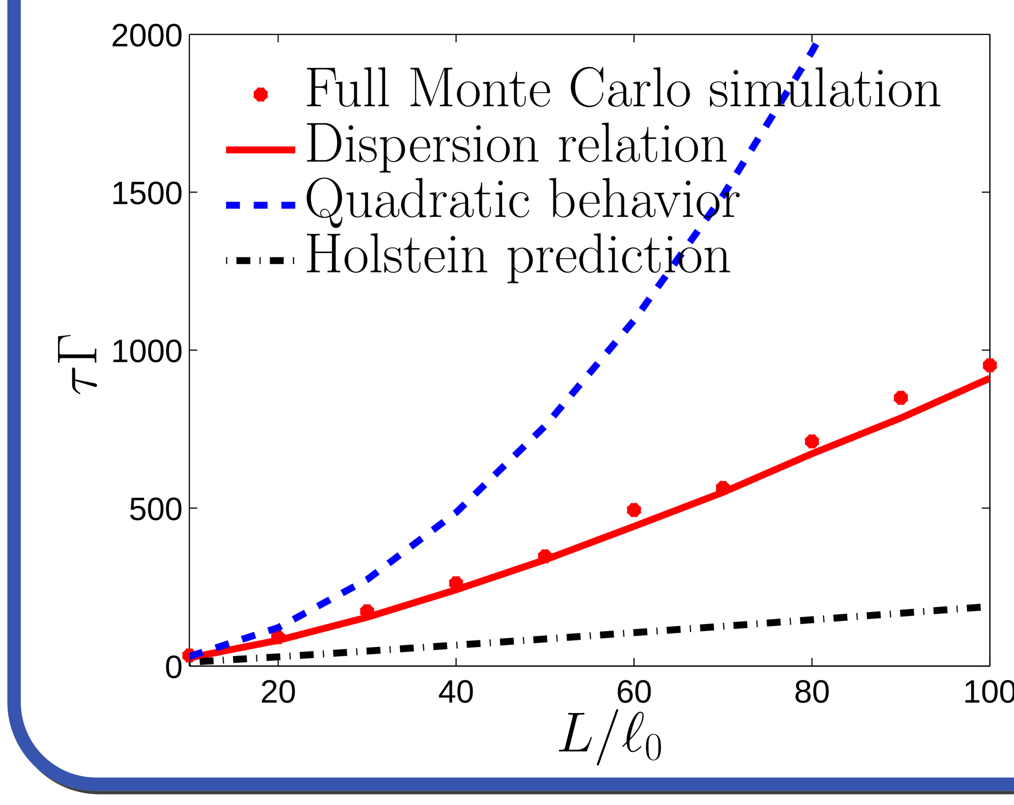
$\boxed{\exp[-N^3/3\left(k_0\bar{v}/\Gamma\right)^2]}$ [4].

4 Modal approach for the diffuson

- **Analytical method:**

$k_0\bar{v}/\Gamma \ll 1$ ⇒ Fokker-Planck equation → Spatial Fourier transform + temporal eigenvalues: $I\left(\mathbf{u},\mathbf{r},\delta,t\right) = g\left(\mathbf{u},\delta\right)\exp\left[i\mathbf{q}\cdot\mathbf{r}+s\left(\mathbf{q}\right)t\right]$ → Preponderant spatial mode: $q = \pi/L$ → Dispersion relation similar to the Schrödinger equation

- **Pinned atoms case**: $\tau = -1/s = 3L^2/\ell_0^2/\left(\pi^2\Gamma\right)$. The diffusion approximation exists [5, 2].
- **Moving atoms case**: we can't define a diffusion coefficient, numerical resolution necessary.



– Behavior very different from the CFR case in which

$$\tau \propto L\sqrt{\ln\left(L/\left(2l_s\left(\omega_0\right)\right)\right)}\ [5];$$

– We can discrimate between the quadratic and the Doppler regimes.

References

[1] R. Pierrat, B. Grémaud, and D. Delande. Enhancement of radiation trapping for quasi-resonant scatterers at low temperature. *Submitted to Phys. Rev. A*, 2009.

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[4] G. Labeyrie, D. Delande, R. Kaiser, and M. Miniatura. Light Transport in Cold Atoms and Thermal Decoherence. *Phys. Rev. Lett.*, 97:013004–1–013004–4, 2006.

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