2008-15D
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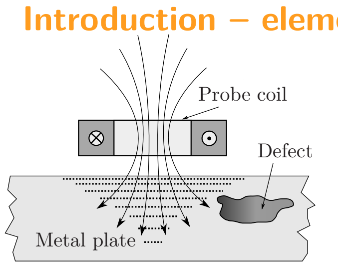
Sándor Bilicz, Marc Lambert, Szabolcs Gyimóthy

A new database generation method combining maximin method and kriging prediction for eddy-current testing

Framework

- PhD thesis in french-hungarian cotutelle (2008-2011)
- Université Paris-Sud 11 and Budapest University of Tecnology and Economics
- French part is financed by Digiteo (project No. 2008-15D)

1 Introduction – elements of ECT



- Probe coil ⇒ alternating magnetic field
- Eddy-currents in the specimen
- Field distortion due to the defect
- Defect $(\mathbf{x})$ ⇒ coil impedance change (ΔZ)
- Forward problem: Configuration and defect properties are known; defect $(\mathbf{x}) \Rightarrow \Delta Z$
- Inverse problem: Measured impedance variations; $\Delta Z \Rightarrow (\mathbf{x})$

2 Motivation

Challenges:

- Forward problem: numerical simulation (Integral equations, FEM,...)
  ⇒ high computational load, "expensive-to-run" simulators
- Inverse problem: solution via several forward simulations
- Required: fast and reliable "direct" method for inversion

Main idea:

Expensive SIMULATION	⇒	Cheap EMULATION
Physical model	⇒	Surrogate model

"Off-line" method: Database

- Pre-calculated results in a database
- Use of the stored data: no field computation is needed any longer
- Speed-up the forward/inverse solutions

3 Database principle and properties

- Input data (simulations and/or measurements) $\mathbf{x} = \{x_i\}, i = 1, \dots, p$ with $\mathbf{x} \in \mathbb{X}$
- Output data (simulations and/or measurements) $\mathbf{y} = \{y_j\}, j = 1, \dots, q$ with $\mathbf{y} \in \mathbb{Y}$

$$\mathbf{y} = \mathcal{F}(\mathbf{x})$$

with $\mathcal{F}$ a numerical model and/or acquisition system ("forward operator")

- Database : L pairs $\left\{(\mathbf{x}^{(l)}, \mathbf{y}^{(l)}),\ l = 1, \dots, L\right\} \in \mathbb{L}$ solutions of $\mathbf{y} = \mathcal{F}(\mathbf{x})$
- Surrogate model: $\tilde{\mathcal{F}}_L(\cdot) \simeq \mathcal{F}(\cdot)$ such as : $\left\|\mathcal{F}(\mathbf{x}) - \tilde{\mathcal{F}}_L(\mathbf{x})\right\|_2 \to$ small , $\forall \mathbf{x} \in \mathbb{X}$

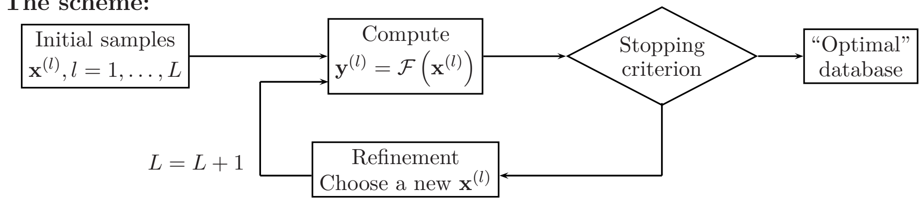
Requirements:

- L as small as possible
  ⇒ *Minimum number of simulations/measurements*
- $\left\{\mathbf{x}^{(l)}\right\}$ filling as good as possible $\mathbb{X}$ – classical "Design-of-Experiment" methods
  ⇒ *Good description of the input* $\mathbb{X}$
- $\left\{\mathbf{y}^{(l)}\right\}$ filling as good as possible $\mathbb{Y}$ – novelty of our approach
  ⇒ *Good description of the output* $\mathbb{Y}$

4 Generation of optimal database by maximin method

- "Distance" definition : $\mathcal{Q}_l(\mathbf{x}) = \left\|\mathcal{F}(\mathbf{x}) - \mathcal{F}\left(\mathbf{x}^{(l)}\right)\right\|_2, \forall \mathbf{x} \in \mathbb{X}$
- Choice of the next point (refinement): $\mathbf{x}^{(l+1)} = \arg\max_{\mathbf{x}\in\mathbb{X}} \left[\min_{l=1,\dots,L} \mathcal{Q}_l(\mathbf{x})\right]$

The scheme:



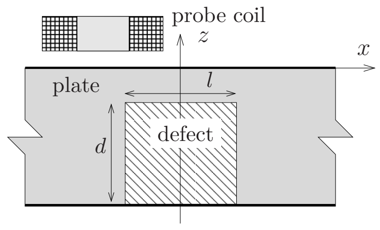
Complex optimization problem to get $\mathbf{x}^{(l+1)}$ – Speeding up by kriging prediction:

- Known : $\mathcal{Q}_k\left(\mathbf{x}^{(l)}\right), l = 1, \dots, L, k = 1, \dots, L$
- Prediction of $\hat{\mathcal{Q}}_k(\mathbf{x}), \forall \mathbf{x} \in \mathbb{X}$ knowing $[\lambda_1(\mathbf{x}), \dots, \lambda_L(\mathbf{x})]^{\mathrm{T}}$ via kriging

$$\begin{bmatrix} \hat{\mathcal{Q}}_1(\mathbf{x}) \\ \vdots \\ \hat{\mathcal{Q}}_L(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} \mathcal{Q}_1(\mathbf{x}^1) & \dots & \mathcal{Q}_1(\mathbf{x}^L) \\ \vdots & \ddots & \vdots \\ \mathcal{Q}_L(\mathbf{x}^1) & \dots & \mathcal{Q}_L(\mathbf{x}^L) \end{bmatrix} \cdot \begin{bmatrix} \lambda_1(\mathbf{x}) \\ \vdots \\ \lambda_L(\mathbf{x}) \end{bmatrix},$$

- Cheaper optimization problem: $\mathbf{x}^{(l+1)} = \arg\max_{\mathbf{x}\in\mathbb{X}} \left[\min_{l=1,\dots,L} \hat{\mathcal{Q}}_l(\mathbf{x})\right]$

5 Illustration – thin crack with 2 parameters

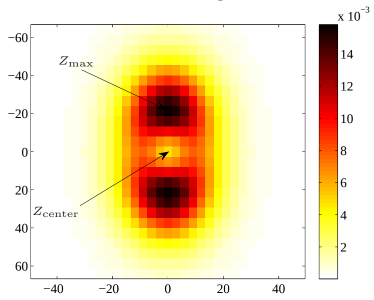


- Coil impedance at $a \times a$ positions (here $a = 29$)
- Defect parameters: length (l), depth (d)

$$\mathbf{x} = [l\ ,\ d]$$

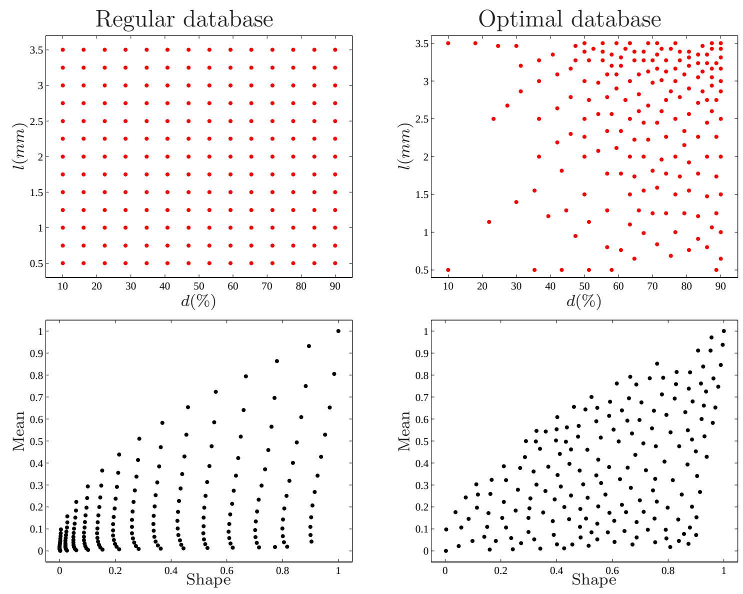
$$0.5\,\mathrm{mm} < l < 3.5\,\mathrm{mm}\ ,\ 10\,\% < d < 90\,\%$$

Measured signal:



Output data:

- $Z_{\mathrm{center}}(\mathbf{x}) = |\Delta Z_{n,n}(\mathbf{x})|, n = \frac{a+1}{2}$, a odd
- $Z_{\max}(\mathbf{x}) = \max_{i,j=1,\dots,a} |\Delta Z_{i,j}(\mathbf{x})|$
- $Z_{\mathrm{mean}}(\mathbf{x}) = \frac{1}{a^2}\sum_{i=1}^{a}\sum_{j=1}^{a} |\Delta Z_{i,j}(\mathbf{x})|$
- $\mathbf{y}(\mathbf{x}) = \left[Z_{\mathrm{mean}}(\mathbf{x}), \frac{Z_{\mathrm{center}}(\mathbf{x})}{Z_{\max}(\mathbf{x})}\right]$



Contact: sandor.bilicz@lss.supelec.fr, marc.lambert@lss.supelec.fr