Derivation and quantization of the surface plasmon field

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Abstract. We focus on the surface plasmon-polaritons (SPP) propagating along flat surfaces. Their propagation is often described in terms of a mode $\exp[i(K_{\parallel} r_{\parallel} + \omega t)]$ a surface wave characterized by a frequency $\omega$ and a wavevector $K_{\parallel}$ parallel to the surface, decaying exponentially in the perpendicular direction. When losses in the materials are taken into account, $K_{\parallel}$ and $\omega$ can no longer be both real. Absorption can be described using a complex $\omega$ or a complex wavevector $K_{\parallel}$. Here we derive two decompositions for the field of surface plasmons to describe their propagation, that can be used to discuss a few concerns concerning SPP. We then deal with a quantum treatment of SPP, which can be used to describe the interaction of a system close to the interface and the surface plasmons of this interface.

Derivation of the surface plasmon field

Using Green’s formulism, we derived in a rigorous way the field of surface plasmons, including its vectorial structure. This method gives explicitly the amplitude of each surface mode as a function of the current $\rho(t)$ exciting them, hence giving the surface plasmon field associated to a distribution of dipoles for instance.

General representation of the field

$E_1(x,y,z,t) = \frac{\mu_0}{2\pi}\int d^2 r' \frac{\rho(r',t)}{|r - r'|} \exp[i(K_{\parallel} r_{\parallel} + \omega t)]$

In the above expression, the wavevector $K_{\parallel}$ is complex, $K_{\parallel} = K_{\parallel}^0 + iK_{\parallel}^{\gamma}$, with $K_{\parallel}^{\gamma}$ the damping wavevector that is always imaginary, $\omega$ is the complex frequency that is also usually complex, $r_{\parallel}$ is parallel to the surface, decaying exponentially in the perpendicular direction. When losses in the materials are taken into account, $K_{\parallel}$ and $\omega$ can no longer be both real. Absorption can be described using a complex $\omega$ or a complex wavevector $K_{\parallel}$. Here we derive two decompositions for the field of surface plasmons to describe their propagation, that can be used to discuss a few concerns concerning SPP.

Quantization of surface electromagnetic waves

Using an expansion over the wavevector $K_{\parallel}$ of the surface wave field, we quantize the field canonically and obtain its operators. This allows us to derive the Einstein coefficients associated to surface plasmons. Although some of the results derived here could have been derived semi-classically, this quantization is more easily generalized to multiple interface surface waves ($\Sigma_{k=1}^N$) than semi-classical methods. Losses in the materials are neglected hence, so that these results are valid only on the left part of the surface plasmon dispersion relation, as we checked it for the case of silver SPP.

Validation on Ag SPP

Graded agreement between the lossless approach and one including losses, except close to the asymptote of the SPP dispersion relation.

Lifetime of a dipole w/ losses

Starting from a description of surface plasmon-polaritons as a sum of modes, we derived the quantum operators acting on the SPP field, our quantum treatment of SPP allows to describe quantum plasmonic effects such as single plasmon interference, quantum coherences or the interaction of SPP with other quantum objects.