



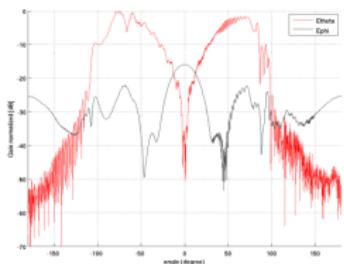
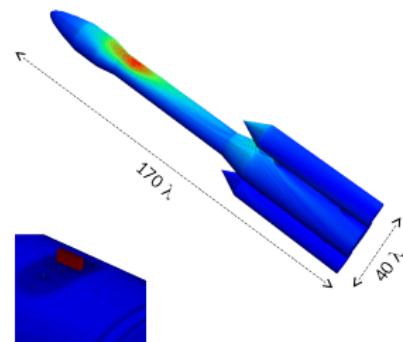
DE LA RECHERCHE À L'INDUSTRIE

Méthode de décomposition de domaine surfacique pour la diffraction d'ondes électromagnétiques

Francis Collino Justine Labat Agnès Pujols | CEA-CESTA

Journée DDM au CESTA - 24 mars 2022

- Context : Solution of **electromagnetic** wave scattering problem by a **complex** object using a boundary element method
- Motivation : Accurate evaluation of **radar cross-sections**
 - ▶ Large-scale objects (in comparison with the wavelength)
 - ▶ Multi-scale phenomena



Example : Monopole antenna in the presence of a dielectric object on a launcher (ISAE Workshop, 2016)

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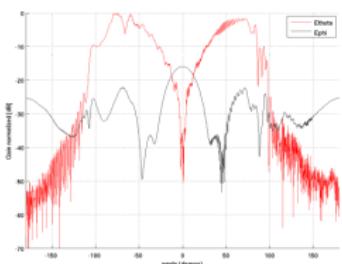
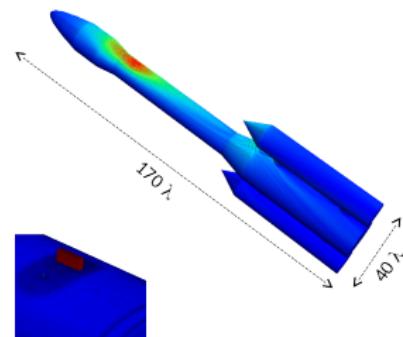
- Motivation : Accurate evaluation of **radar cross-sections**

 - ▶ Large-scale objects (in comparison with the wavelength)
 - ▶ Multi-scale phenomena

- Difficulties : **High computational costs**

 - ▶ Number of degrees of freedom : 1 650 875
→ Approximately 100 000 000 in volume

Solver	Number of CPU	Factorization (in CPU·H)
LL ^t (prediction)	15 000	151 435
Block low-rank	800	2 541
H-matrix	200	1 043



Example : Monopole antenna in the presence of a dielectric object on a launcher (ISAE Workshop, 2016)

Hackbush (1999), Bebendorf (2008), ...

- Context : Solution of **electromagnetic** wave scattering problem by a **complex** object using a boundary element method

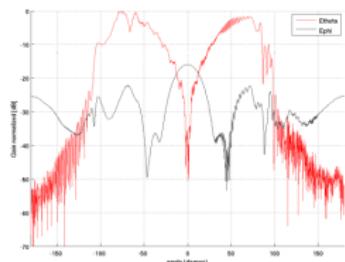
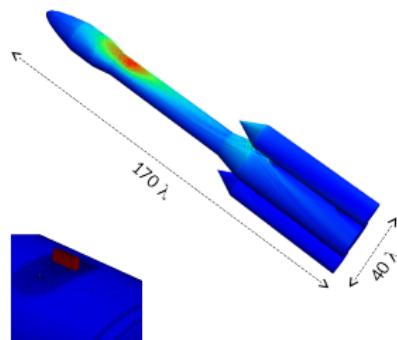
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- Objectives : Development of a **domain decomposition** method

- ▶ Robust simulations for a **wide range** of frequencies
 - Solve very large problems (more than 10 000 000 of unknowns)
 - ▶ **Geometry-adaptative** strategies to handle multi-scale structures
 - Allow non-conformal meshes
 - ▶ Simulation-based engineering using **High Performance Computing**



Example : Monopole antenna in the presence of a dielectric object on a launcher (ISAE Workshop, 2016)

- Adaptation of volumical **discontinuous Galerkin** methods to surface problems

Peng, Hiptmair and Shao (2016), Messai and Pernet (2020)

	Volume	Surface	Comments
Locality	✓	✗	Iterative solution is inevitable
Restrictions	✓	✗	Restrictions of distributions are not easy to define
Traces	✓	✗	No trace theorem in $H_t^{-\frac{1}{2}}(\text{div}_\Gamma, \Gamma)$

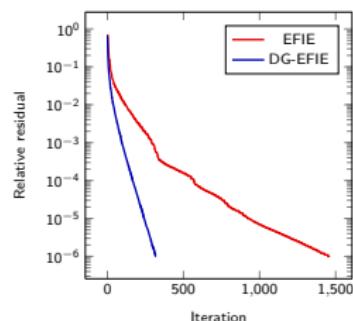
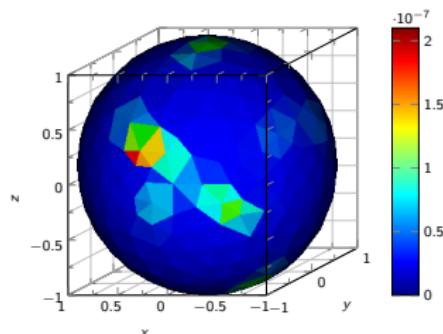
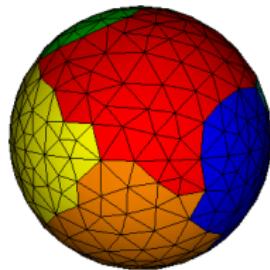
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- Why not to directly use an iterative solver on BEM?

- Main motivation : **non-conformal** meshes
- EFIE simulation preconditioned using domain partitioning on a low-frequency case



- 1 Discontinuous Galerkin-based surface domain decomposition method**
 - Model problem
 - Discontinuous formulation
 - Discrete formulation
 - Iterative procedure
- 2 Numerical results**
 - Accuracy of solutions
 - Convergence of the iterative solver
 - Computational costs
 - The non-conformal case
- 3 Conclusion and perspectives**

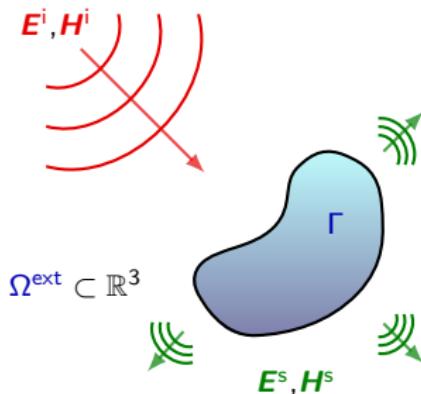
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3 Conclusion and perspectives



- Using a time-harmonic dependence in $\exp(+i\omega t)$

$$\nabla \times \mathbf{E}^s + i\kappa Z_0^{-1} \mathbf{H}^s = 0 \quad \text{in } \Omega^{\text{ext}}$$

$$\nabla \times \mathbf{H}^s - i\kappa Z_0 \mathbf{E}^s = 0 \quad \text{in } \Omega^{\text{ext}}$$

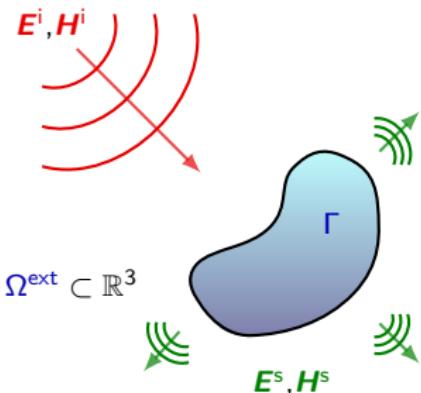
$$\mathbf{n} \times \mathbf{E}^s = -\mathbf{n} \times \mathbf{E}^i \quad \text{on } \Gamma$$

$$\lim_{|x| \rightarrow \infty} |x| (Z_0 \mathbf{H}^s \times \hat{x} - \mathbf{E}^s) = 0 \quad \text{unif. in } \hat{x} = \frac{x}{|x|}$$

κ : wave-number, Z_0 : impedance coefficient in vacuum

- Ω^{ext} : Exterior domain
- Γ : Scattering surface
- $\mathbf{E}^i, \mathbf{H}^i$: Incident fields
- $\mathbf{E}^s, \mathbf{H}^s$: Scattered fields
- \mathbf{E}, \mathbf{H} : Total fields

$$\mathbf{E} = \mathbf{E}^i + \mathbf{E}^s \quad \mathbf{H} = \mathbf{H}^i + \mathbf{H}^s$$



- Ω^{ext} : Exterior domain
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- Using a time-harmonic dependence in $\exp(+i\omega t)$

$$\nabla \times E^s + i\kappa Z_0^{-1} H^s = 0 \quad \text{in } \Omega^{\text{ext}}$$

$$\nabla \times H^s - i\kappa Z_0 E^s = 0 \quad \text{in } \Omega^{\text{ext}}$$

$$n \times E^s = -n \times E^i \quad \text{on } \Gamma$$

$$\lim_{|x| \rightarrow \infty} |x| (Z_0 H^s \times \hat{x} - E^s) = 0 \quad \text{unif. in } \hat{x} = \frac{x}{|x|}$$

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- Using the Stratton-Chu formulas for $x \in \Omega^{\text{ext}}$

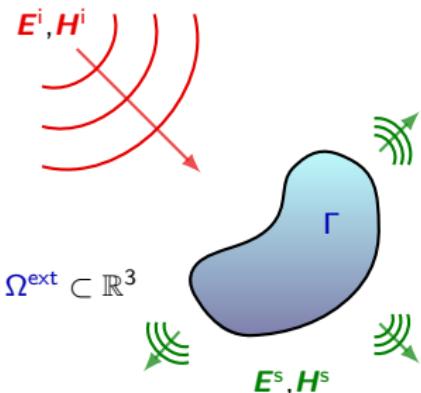
$$E^s(x) = -i\kappa \mathcal{T}J(x) \quad \text{and} \quad H^s(x) = \frac{1}{Z_0} \mathcal{K}J(x)$$

$$J = Z_0(n \times H) : \text{total surface electric current}$$

$$\mathcal{T}J = \frac{1}{\kappa^2} \nabla (\mathcal{S} \operatorname{div}_\Gamma J) + \mathcal{S}J$$

$$\mathcal{K}J = \nabla \times \mathcal{S}J \quad \tilde{\mathcal{S}}\lambda(x) = \int_\Gamma G(x, y)\lambda(y) ds_y$$

G : out-going Green function of the Helmholtz equation



- Ω^{ext} : Exterior domain
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κ : wave-number, Z_0 : impedance coefficient in vacuum

- Using the Stratton-Chu formulas for $x \in \Omega^{\text{ext}}$

$$E^s(x) = -i\kappa TJ(x) \quad \text{and} \quad H^s(x) = \frac{1}{Z_0} KJ(x)$$

$J = Z_0(\mathbf{n} \times \mathbf{H})$: total surface electric current

- Jump relations give boundary integral equations

$$i\kappa TJ = \mathbf{n} \times (E^i \times \mathbf{n}) \quad \text{on } \Gamma \quad (\text{EFIE})$$

$$\frac{1}{2} J - KJ = Z_0(\mathbf{n} \times H^i) \quad \text{on } \Gamma \quad (\text{MFIE})$$

$$i\kappa T \mathbf{J} = \mathbf{n} \times (\mathbf{E}^i \times \mathbf{n}) \quad \text{on } \Gamma \quad (\mathbf{EFIE})$$

$$\frac{1}{2} \mathbf{J} - K \mathbf{J} = Z_0(\mathbf{n} \times \mathbf{H}^i) \quad \text{on } \Gamma \quad (\mathbf{MFIE})$$

$$T : \mathbf{H}_t^{-\frac{1}{2}}(\text{div}_\Gamma, \Gamma) \longrightarrow \mathbf{H}_t^{-\frac{1}{2}}(\text{curl}_\Gamma, \Gamma)$$

$$T \mathbf{J} = \frac{1}{\kappa^2} \nabla_\Gamma (S \text{div}_\Gamma \mathbf{J}) + S \mathbf{J}$$

$$K : \mathbf{H}_t^{-\frac{1}{2}}(\text{div}_\Gamma, \Gamma) \longrightarrow \mathbf{H}_t^{-\frac{1}{2}}(\text{div}_\Gamma, \Gamma)$$

$$K \mathbf{J} = \text{p.v.} [\mathbf{n} \times (\nabla_\Gamma \times S \mathbf{J})]$$

$$i\kappa \mathbf{T}\mathbf{J} = \mathbf{n} \times (\mathbf{E}^i \times \mathbf{n}) \quad \text{on } \Gamma \quad (\mathbf{EFIE})$$

$$\frac{1}{2}\mathbf{J} - \mathbf{K}\mathbf{J} = Z_0(\mathbf{n} \times \mathbf{H}^i) \quad \text{on } \Gamma \quad (\mathbf{MFIE})$$

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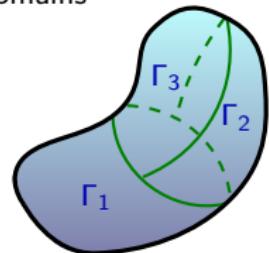
$$\mathbf{K}\mathbf{J} = \text{p.v.} [\mathbf{n} \times (\nabla_\Gamma \times S\mathbf{J})]$$

- N : number of subdomains

- $\Gamma = \bigcup_{n=1}^N \Gamma_n$

- $\gamma_{nm} = \Gamma_n \cap \Gamma_m$

- $\gamma_n = \partial\Gamma_n$



- τ_{nm} : exterior normal vector to γ_{nm}

- τ_n : exterior normal vector to γ_n

$$i\kappa \sum_{m=1}^N T_{nm} \mathbf{J}_m = \mathbf{n} \times (\mathbf{E}^i \times \mathbf{n}) \quad \text{on } \Gamma_n$$

$$T_{nm} : \widetilde{\mathbf{H}}_t^{-\frac{1}{2}}(\operatorname{div}_{\Gamma_m}, \Gamma_m) \longrightarrow \widetilde{\mathbf{H}}_t^{-\frac{1}{2}}(\operatorname{curl}_{\Gamma_n}, \Gamma_n)$$

$$\mathbf{T}_{nm} \mathbf{J}_m = \frac{1}{\kappa^2} \nabla_{\Gamma_n} (S_{nm} \operatorname{div}_{\Gamma_m} \mathbf{J}_m) + S_{nm} \mathbf{J}_m$$

1 Restriction to each subdomain

$$\mathbf{J}_m = \mathbf{J}|_{\Gamma_m} \in \widetilde{\mathbf{H}}_t^{-\frac{1}{2}}(\operatorname{div}_{\Gamma_m}, \Gamma_m)$$

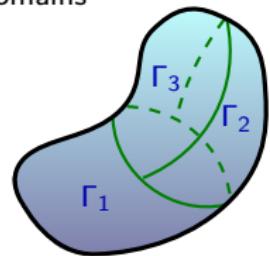
$$\frac{1}{2} \mathbf{J}_n - \sum_{m=1}^N K_{nm} \mathbf{J}_m = Z_0(\mathbf{n} \times \mathbf{H}^i) \quad \text{on } \Gamma_n$$

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1 Restriction to each subdomain

$$\mathbf{J}_m = J|_{\Gamma_m} \in \widetilde{\mathbf{H}}_t^{-\frac{1}{2}}(\operatorname{div}_{\Gamma_m}, \Gamma_m)$$

2 Obtention of variational formulations

- ▶ Multiplication by a test-function
- ▶ Integration on partial surfaces
- ▶ Integration by parts (EFIE only)

$$\begin{aligned} & \langle \nabla_{\Gamma} (S_{nm} \operatorname{div}_{\Gamma} \mathbf{J}_m), \mathbf{v}_n \rangle_{\Gamma_n} = \\ & - \langle S_{nm} \operatorname{div}_{\Gamma} \mathbf{J}_m, \operatorname{div}_{\Gamma} \mathbf{v}_n \rangle_{\Gamma_n} + \langle S_{nm} \operatorname{div}_{\Gamma} \mathbf{J}_m, \mathbf{v}_n \cdot \boldsymbol{\tau}_n \rangle_{\gamma_n} \end{aligned}$$

▶ which requires **more regularity** ($\varepsilon > 0$)

$$\operatorname{div}_{\Gamma} \mathbf{J}_m \in H^{-\frac{1}{2}+\varepsilon}(\Gamma_m) \text{ and } \mathbf{v}_n \in \mathbf{H}_t^{\frac{1}{2}-\varepsilon}(\operatorname{div}_{\Gamma}, \Gamma_n)$$

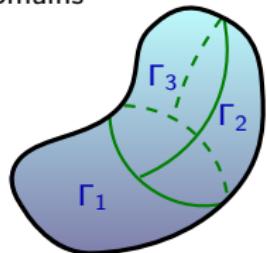
$$\frac{1}{2} \mathbf{J}_n - \sum_{m=1}^N K_{nm} \mathbf{J}_m = Z_0(\mathbf{n} \times \mathbf{H}^i) \quad \text{on } \Gamma_n$$

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$$\mathbf{J}_m = J|_{\Gamma_m} \in \widetilde{\mathbf{H}}_t^{-\frac{1}{2}}(\operatorname{div}_{\Gamma_m}, \Gamma_m)$$

2 Obtention of variational formulations

- ▶ Multiplication by a test-function
- ▶ Integration on partial surfaces
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3 Summation over all the subdomains

$$\sum_{n=1}^N \langle S_{nm} \operatorname{div}_{\Gamma} \mathbf{J}_m, \mathbf{v}_n \cdot \boldsymbol{\tau}_n \rangle_{\gamma_n} =$$

$$\sum_{\gamma_{nm}} \langle S_{nm} \operatorname{div}_{\Gamma} \mathbf{J}_m, [\mathbf{v}]_{\gamma_{nm}} \rangle_{\gamma_{nm}}$$

$$[\mathbf{v}]_{\gamma_{nm}} = \boldsymbol{\tau}_{nm} \cdot \mathbf{v}_m + \boldsymbol{\tau}_{mn} \cdot \mathbf{v}_n : \text{jump across } \gamma_{nm}$$

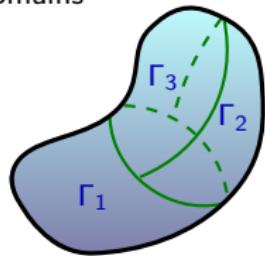
$$\frac{1}{2} \mathbf{J}_n - \sum_{m=1}^N K_{nm} \mathbf{J}_m = Z_0 (\mathbf{n} \times \mathbf{H}^i) \quad \text{on } \Gamma_n$$

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Find $\mathbf{J} \in \bigoplus_{m=1}^N \mathbf{H}_t^{-\frac{1}{2}+\varepsilon}(\text{div}_{\Gamma_m}, \Gamma_m)$ with $\varepsilon > 0$ such that

$$a_{\Gamma}(\mathbf{J}, \mathbf{v}) + a_{\gamma}^{\pm}(\mathbf{J}, \mathbf{v}) + p_{\gamma}^*(\mathbf{J}, \mathbf{v}) = \ell(\mathbf{v}) \quad \text{for any } \mathbf{v} \in \bigoplus_{n=1}^N \mathbf{H}_t^{\frac{1}{2}-\varepsilon}(\text{div}_{\Gamma_n}, \Gamma_n)$$

$$a_{\Gamma}(\mathbf{J}, \mathbf{v}) = \sum_{n=1}^N \sum_{m=1}^N \left\{ \frac{1}{i\kappa} \langle S_{nm} \text{div}_{\Gamma_m} \mathbf{J}_m, \text{div}_{\Gamma_n} \mathbf{v}_n \rangle_{\Gamma_n} + i\kappa \langle S_{nm} \mathbf{J}_m, \mathbf{v}_n \rangle_{\Gamma_n} \right\} \quad (\text{continuous EFIE})$$

$$a_{\gamma}^{\pm}(\mathbf{J}, \mathbf{v}) = -\frac{1}{i\kappa} \sum_{m=1}^N \sum_{\gamma_{nm}} \langle S_{nm} \text{div}_{\Gamma_m} \mathbf{J}_m, [\mathbf{v}]_{\gamma_{nm}} \rangle_{\gamma_{nm}} + \tilde{a}_{\gamma}^{\pm}(\mathbf{J}, \mathbf{v}) \quad (\text{symmetrization})$$

$$p_{\gamma}^*(\mathbf{J}, \mathbf{v}) \text{ is function of } [\mathbf{J}]_{\gamma_{nm}} \text{ and } [\mathbf{v}]_{\gamma_{nm}} \quad (\text{penalization})$$

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$p_{\gamma}^*(\mathbf{J}, \mathbf{v})$ is function of $[\mathbf{J}]_{\gamma_{nm}}$ and $[\mathbf{v}]_{\gamma_{nm}}$ (penalization)

$$\tilde{a}_{\gamma}^{-}(\mathbf{J}, \mathbf{v}) = -\frac{1}{i\kappa} \sum_{n=1}^N \sum_{\gamma_{mn}} \langle S_{mn} \text{div}_{\Gamma_n} \mathbf{v}_n, [\mathbf{J}]_{\gamma_{mn}} \rangle_{\gamma_{mn}} \quad (\mathbf{S})$$

$$\tilde{a}_{\gamma}^{+}(\mathbf{J}, \mathbf{v}) = +\frac{1}{i\kappa} \sum_{n=1}^N \sum_{\gamma_{mn}} \langle S_{mn} \text{div}_{\Gamma_n} \mathbf{v}_n, [\mathbf{J}]_{\gamma_{mn}} \rangle_{\gamma_{mn}} \quad (\mathbf{AS})$$

► Requires more regularity ($\varepsilon = \frac{1}{2}$)

$$\mathbf{J} \text{ and } \mathbf{v} \in \bigoplus_{n=1}^N \mathbf{L}_t^2(\text{div}_{\Gamma_n}, \Gamma_n)$$

Find $\mathbf{J} \in \bigoplus_{m=1}^N \mathbf{L}_t^2(\text{div}_{\Gamma_m}, \Gamma_m)$ such that

$$a_\Gamma(\mathbf{J}, \mathbf{v}) + a_\gamma^*(\mathbf{J}, \mathbf{v}) + p_\gamma^*(\mathbf{J}, \mathbf{v}) = \ell(\mathbf{v}) \quad \text{for any } \mathbf{v} \in \bigoplus_{n=1}^N \mathbf{L}_t^2(\text{div}_{\Gamma_n}, \Gamma_n)$$

- Empirical choice coming from Peng, Hiptmair and Shao (2016) : $L^2(\gamma)$ -inner product

$$p_\gamma^0(\mathbf{J}, \mathbf{v}) = \frac{\beta}{\kappa} \sum_{\gamma_{nm}} \langle [\mathbf{J}]_{\gamma_{nm}}, [\mathbf{v}]_{\gamma_{nm}} \rangle_{\gamma_{nm}} \quad (\mathbf{L}^2)$$

where β have to be without dimension

► requires **more regularity** (one more time !) to make sense : \mathbf{J} and $\mathbf{v} \in \bigoplus_{n=1}^N \mathbf{H}_t^{\frac{1}{2}}(\text{div}_{\Gamma_n}, \Gamma_n)$

- A new penalization : $H^{-\frac{1}{2}}(\gamma)$ -inner product (positive definite bilinear form)

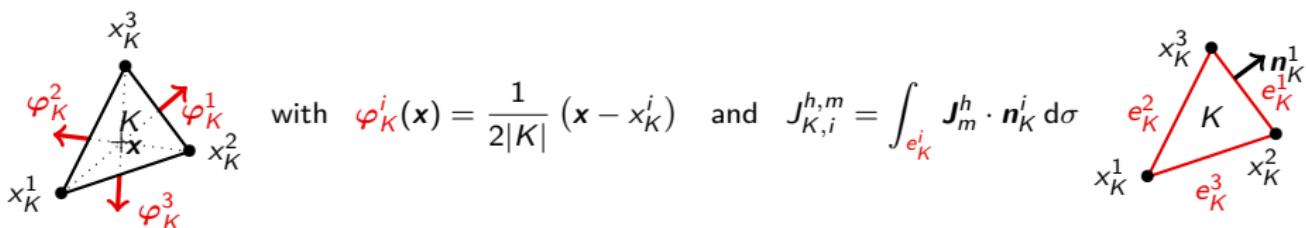
$$p_\gamma^{-\frac{1}{2}}(\mathbf{J}, \mathbf{v}) = \beta \sum_{\gamma_{nm}} \langle S_{\gamma_{nm}} [\mathbf{J}]_{\gamma_{nm}}, [\mathbf{v}]_{\gamma_{nm}} \rangle_{\gamma_{nm}} \quad (\mathbf{H}^{-\frac{1}{2}})$$

where $\tilde{S}_\gamma \lambda(\mathbf{x}) = \frac{1}{2\pi} \int_{\gamma} K_0(\kappa|\mathbf{x} - \mathbf{y}|) \lambda(\mathbf{y}) d\sigma_{\mathbf{y}}$ and β have to be without dimension

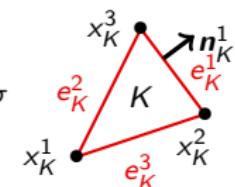
- Discretization space : Restrictions of Raviart-Thomas boundary elements of smallest degree

$$\mathbf{J} \approx \mathbf{J}^h \in \mathcal{V}^h \iff \mathbf{J}^h = \sum_{m=1}^N \mathbf{J}_m^h \quad \mathbf{J}_m^h \in \mathcal{V}_m^h \subset \mathbf{L}_t^2(\operatorname{div}_{\Gamma_m}, \Gamma_m)$$

where $\mathbf{J}_m^h(x) = \sum_{K \in \Gamma_m^h} J_{K,1}^{h,m} \varphi_K^1(x) + J_{K,2}^{h,m} \varphi_K^2(x) + J_{K,3}^{h,m} \varphi_K^3(x)$ $x \in \Gamma_m^h$ (triangulation of Γ_m)



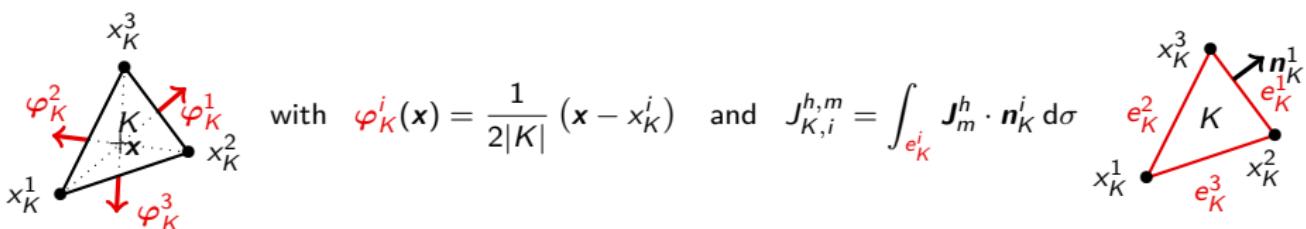
$$\text{with } \varphi_K^i(x) = \frac{1}{2|K|} (x - x_K^i) \quad \text{and} \quad J_{K,i}^{h,m} = \int_{e_K^i} \mathbf{J}_m^h \cdot \mathbf{n}_K^i \, d\sigma$$



- Discretization space : Restrictions of Raviart-Thomas boundary elements of smallest degree

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Find $\mathbf{J}^h \in \mathcal{V}^h$ such that for any $\mathbf{v}^h \in \mathcal{V}^h$

$$a_\Gamma^h(\mathbf{J}^h, \mathbf{v}^h) + a_\gamma^{*,h}(\mathbf{J}^h, \mathbf{v}^h) + p_\gamma^{*,h}(\mathbf{J}^h, \mathbf{v}^h) = \ell^h(\mathbf{v}^h) \quad (\mathbf{DG-EFIE})$$

$$b_\Gamma^h(\mathbf{J}^h, \mathbf{v}^h) = \tilde{\ell}^h(\mathbf{v}^h) \quad (\text{from continuous MFIE}) \quad (\mathbf{DG-MFIE})$$

$$\alpha (\mathbf{DG-EFIE}) + (1 - \alpha) (\mathbf{DG-MFIE}) \quad (\text{with } 0 < \alpha < 1) \quad (\mathbf{DG-CFIE})$$

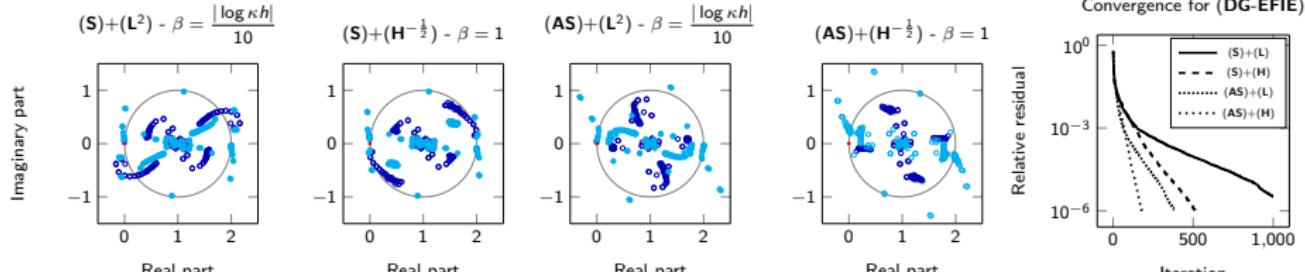
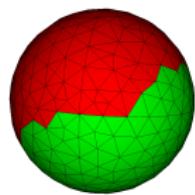
- Use of **GMRes solver** (from CERFACS) with **block-diagonal Jacobi preconditioning**
 - ▶ Diagonal blocks correspond to matrices for individual subdomain
 - Flexibility in choosing subdomain solvers (LL^t , LU, Block low-rank, H-matrix, . . .)
 - ▶ Off-diagonal blocks correspond to interactions between subdomains
 - Could be sped-up by compression techniques

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- Eigenvalue distribution for a low frequency case (at 95 MHz)

- ▶ **(DG-CFIE)** simulation with CFIE parameter $\alpha = 0.5$
- ▶ **(DG-EFIE)** simulation and convergence using `restart = 10`



1 Discontinuous Galerkin-based surface domain decomposition method

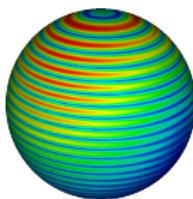
- Model problem
- Discontinuous formulation
- Discrete formulation
- Iterative procedure

2 Numerical results

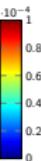
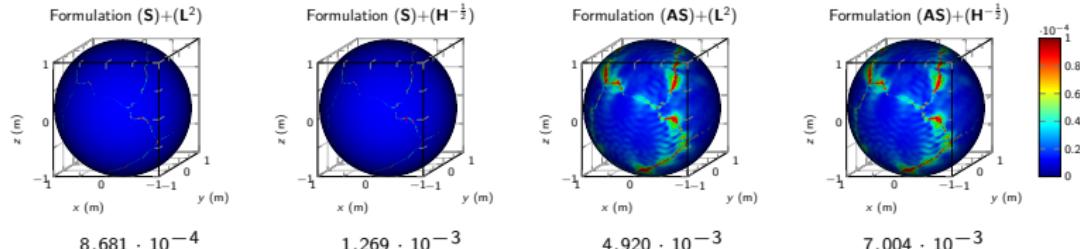
- Accuracy of solutions
- Convergence of the iterative solver
- Computational costs
- The non-conformal case

3 Conclusion and perspectives

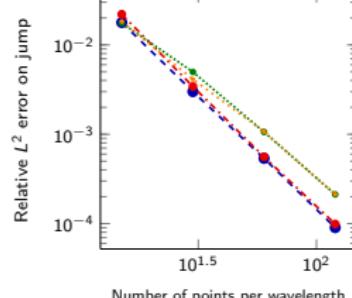
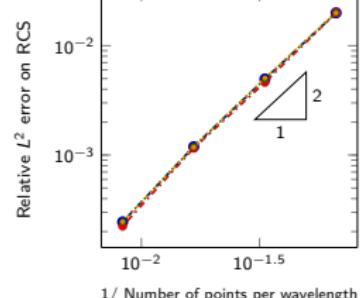
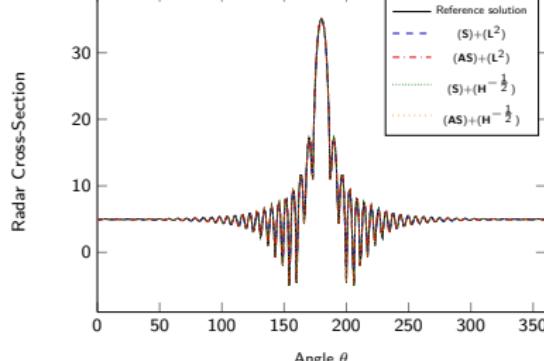
- Difference modulus between electric currents : **(DG-CFIE)** at 1.52 GHz - $h = \frac{1}{15}$



Maximal errors :



- Comparison of associated radar cross-sections and convergence (95 MHz - 2 subdomains)



- Convergence of the iterative solver on the PEC parallelepiped of dimension $1\text{m} \times 1\text{m} \times 0.25\text{m}$ at 5 GHz enlightened with angle $(\frac{\pi}{2}, \frac{\pi}{4})$ with $h_{\text{moy}} = \frac{\lambda}{11}$ and using 8 subdomains

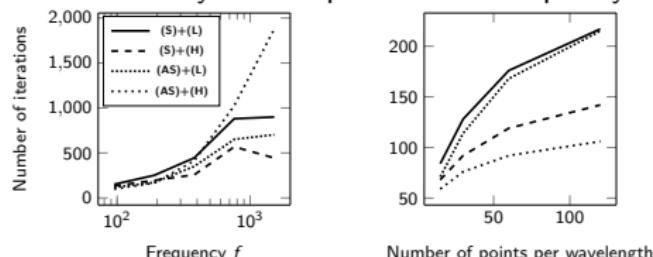
- (**DG-CFIE**) with $\alpha = 0.5$: GMRes without restart and relative residual $\varepsilon = 10^{-3}$

Formulation	$(S) + (L^2)$	$(AS) + (L^2)$	$(S) + (H^{-\frac{1}{2}})$	$(AS) + (H^{-\frac{1}{2}})$
Jump error	$2.32 \cdot 10^{-3}$	$3.13 \cdot 10^{-3}$	$1.01 \cdot 10^{-2}$	$1.03 \cdot 10^{-2}$
RCS error	$2.04 \cdot 10^{-2}$	$3.37 \cdot 10^{-2}$	$1.87 \cdot 10^{-2}$	$3.58 \cdot 10^{-2}$
Number of iterations	14	11	32	16

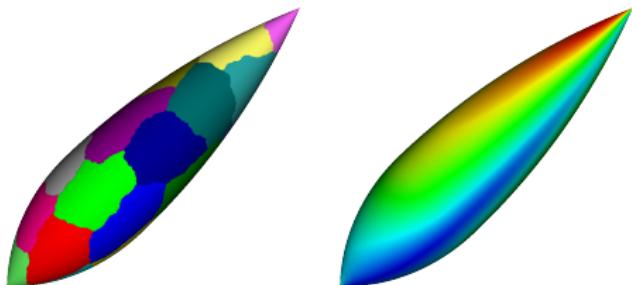
- (**DG-EFIE**) formulation : GMRes with restart = 50 and relative residual $\varepsilon = 10^{-3}$

Formulation	$(S) + (L^2)$	$(AS) + (L^2)$	$(S) + (H^{-\frac{1}{2}})$	$(AS) + (H^{-\frac{1}{2}})$
Jump error	$1.87 \cdot 10^{-3}$	$2.59 \cdot 10^{-3}$	$1.56 \cdot 10^{-3}$	-
RCS error	$6.88 \cdot 10^{-3}$	$6.64 \cdot 10^{-2}$	$3.73 \cdot 10^{-3}$	-
Number of iterations	434	374	269	-

- Scalability with respect to the frequency and the mesh size (without restart and $\varepsilon = 10^{-6}$)



- No significant improvement of $H^{-\frac{1}{2}}$ penalisation for (**DG-CFIE**) formulation
- Improvement of convergence and accuracy for the symmetric (**DG-EFIE**) formulation
- Keeping a symmetric EFIE formulation has benefits for the solver



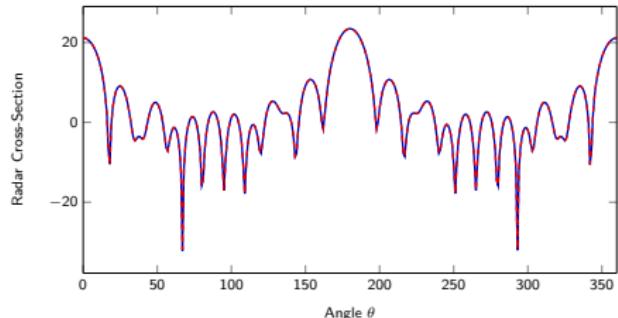
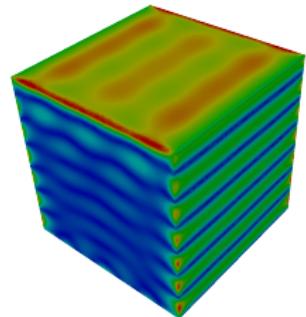
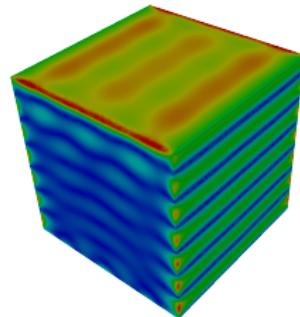
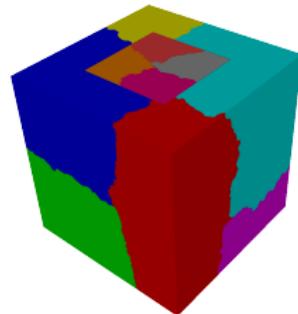
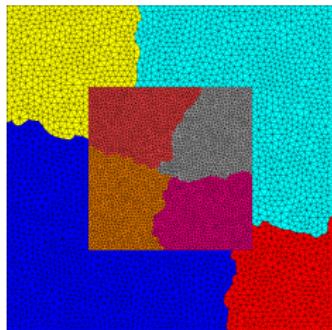
- Dimensions : $10\lambda \times 10\lambda \times 40\lambda$
- Number of degrees of freedom : 273 312
- Additional degrees of freedom : 3 021
- Number of subdomains : 24
- (DG-CFIE) simulation with $\alpha = 0.9$
- Formulation (AS)+(L²)
- GMRes with restart = 50

- Comparison between CFIE+LU, CFIE+GMRes and DG-CFIE+GMRes

Formulation	Solver	Number of CPU	Factorization (in CPU·H)
CFIE	LU	1 280	5 018
DG-CFIE	GMRes	1 280	36

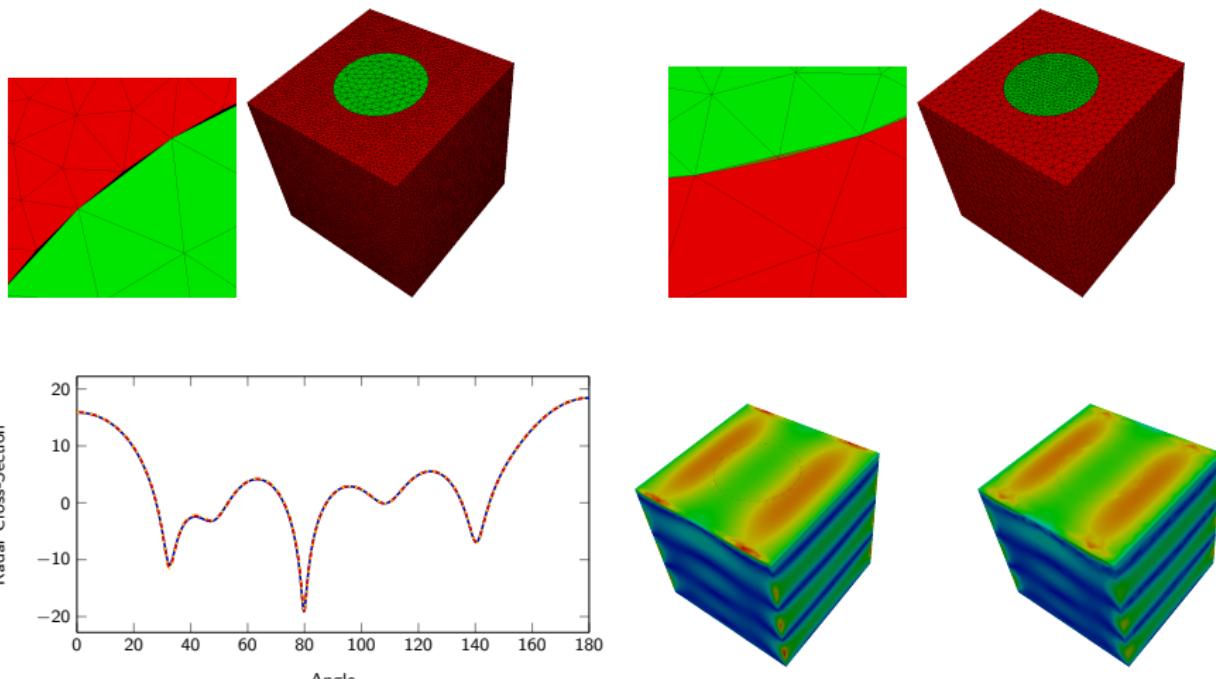
Formulation	Solver	Relative residual	Number of iterations	Convergence (in CPU·H)
CFIE	GMRes	10^{-3}	202	5 697
DG-CFIE	GMRes	10^{-3}	47	1 142
DG-CFIE	GMRes	10^{-6}	150	3 556

- Comparison of electric currents on the PEC cube of side 1 m : Conformal mesh vs. non-conformal mesh at 1 GHz with $h_{\max} = \frac{\lambda}{10}$ and using 10 sub-domains



Mesh	Jump error	RCS error
Refined	$1.19 \cdot 10^{-3}$	$1.79 \cdot 10^{-4}$
Coarse	$3.30 \cdot 10^{-3}$	$3.27 \cdot 10^{-3}$
Non-conformal	$3.42 \cdot 10^{-3}$	$3.26 \cdot 10^{-3}$

- Comparison of electric currents on the PEC cube of side 1 m : Non-conformal (coarse) meshes at 500 MHz using 2 sub-domains



► How to define correctly penalization function on this kind of interfaces ? On-going work ...

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Conclusion

- Development and numerical analysis of a discontinuous Galerkin surface domain decomposition method for electromagnetic scattering by non-penetrable objects
 - ▶ Comparison of **symmetric** and **anti-symmetric** formulations
 - Symmetric formulation involves **better accuracy** on currents and **preserves symmetry** in EFIE
 - Anti-symmetric formulation involves **better conditioning number** in CFIE formulation
 - ▶ Comparison of L^2 and $H^{-\frac{1}{2}}$ interior penalty terms
 - L^2 penalization is robust with respect to the **frequency** but **parameter β** have to be calibrated
 - $H^{-\frac{1}{2}}$ penalization is more robust with respect to the **discretization size**
- Comparison with a boundary element method
 - ▶ Computational costs
 - Using a direct solver : **large gain** in factorization time
 - Using a preconditioned iterative solver : **faster convergence** for EFIE formulation
 - ▶ Accuracy : Jump **does not pollute** radar cross-sections

Perspectives

- Correction of penalization functions to take into account **holes** and **overlaps**
- Integration of **H-matrix** formalism to consider bigger problems
- Extension to **dielectric** scatterers to consider more complex problems

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Thank you for your attention !