A Well-conditioned Weak Coupling of Boundary Element and High-order Finite Element Methods for Time-harmonic Electromagnetic Scattering

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[3]Thales Defense Mission Systems

March 24, 2022

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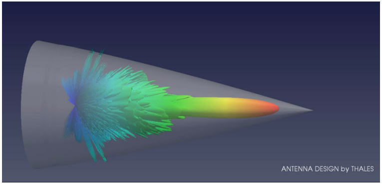
Outline



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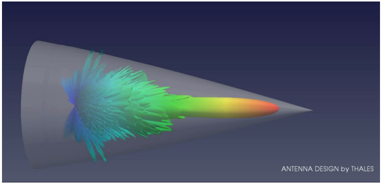
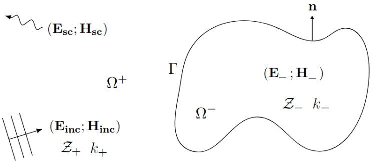


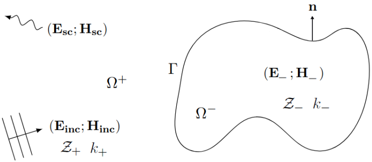
Electromagnetic Scattering Problem



Electromagnetic Scattering Problem

Incident plane wave

$\mathbf{E_{inc}} = (e^{ik_+ \cdot z}; 0; 0)^T$



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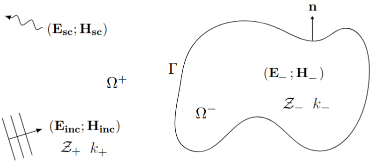
$(\mathbf{E}_+; \mathbf{H}_+) = (\mathbf{E_{inc}}; \mathbf{H_{inc}}) + (\mathbf{E_{sc}}; \mathbf{H_{sc}})$



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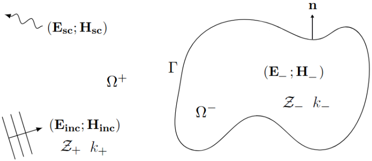
Maxwell harmonic system in Ω

$$\begin{aligned} \mathbf{curl\ E} \quad & k \ Z \ \mathbf{H} = \mathbf{0} \\ \mathbf{curl\ H} \ + & k \ Z \ ^1\mathbf{E} = \mathbf{0} \end{aligned}$$



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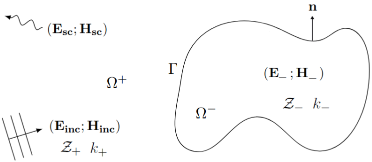
Silver-Müller radiation condition

$Z_+ \mathbf{H}_{\mathbf{sc}} \quad \frac{\mathbf{x}}{jj\mathbf{x}jj} \quad \mathbf{E}_{\mathbf{sc}} = \underset{r \ ! \ +1}{O} (r \quad ^2)$



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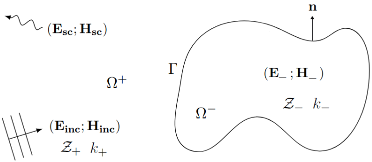
Transmission conditions on Γ

$$\begin{aligned} \mathbf{E}_{\ j\Gamma} \quad \mathbf{n} &= \mathbf{E}_{\mathbf{sc}_{j\Gamma}} \quad \mathbf{n} + \mathbf{E}_{\mathbf{inc}_{j\Gamma}} \quad \mathbf{n} \\ \mathbf{H}_{\ j\Gamma} \quad \mathbf{n} &= \mathbf{H}_{\mathbf{sc}_{j\Gamma}} \quad \mathbf{n} + \mathbf{H}_{\mathbf{inc}_{j\Gamma}} \quad \mathbf{n} \end{aligned}$$

with $\mathbf{n}$ the outward pointing normal on



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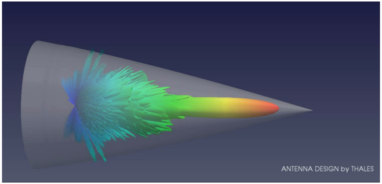
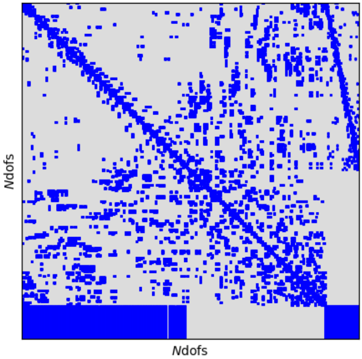
! Two standard approaches to overcome this difficulty:

- Truncation methods (PML or ABC): approximate the scattering problems at the continuous level.
- **Surface integral equations for the exterior domain**: exact at the continuous level and adapted to homogeneous domain.

Outline



Strong Coupling, a Standard Resolution Method



Matrix resulting from the coupling of variational formulation for Ω and an integral equation for Ω_+ in one unique formulation.

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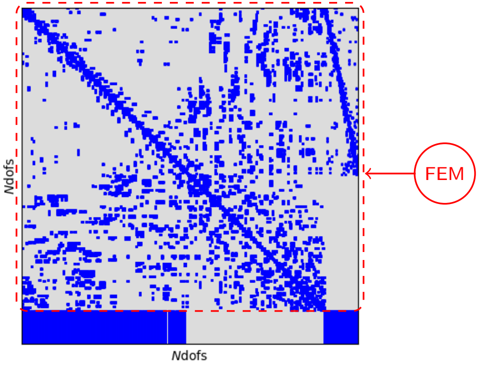

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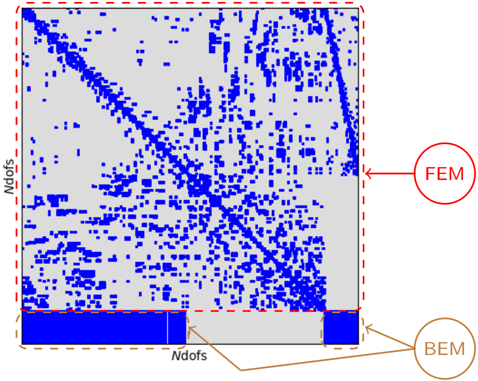

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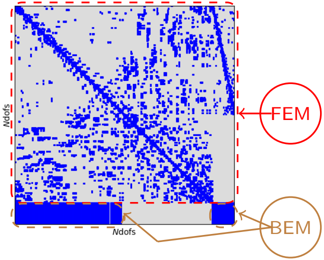
Difficulty for classical iterative methods

Strong FEM-BEM leads to a matrix with sparse and dense blocks.



Motivations for a New Method

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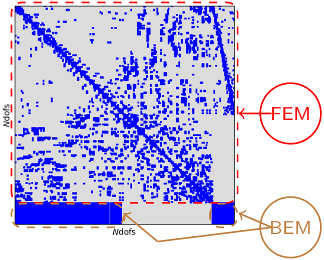
Large problems

How does one combine compression methods (H-matrices...) for the BEM part and DDM for the FEM part?



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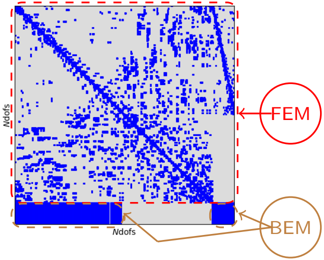
What if you just want to run the pre-existing solvers which are independent and optimized? (could be very technical...)



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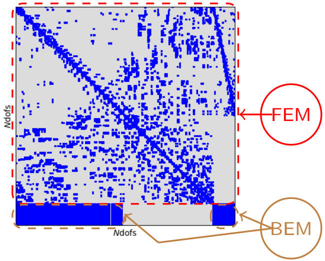

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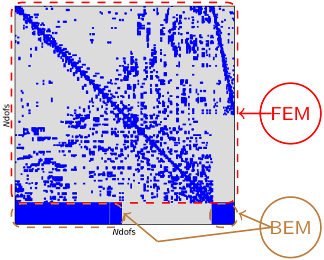
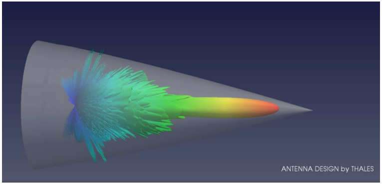


A simple solution to all these problems

Domain decomposition between the interior domain and the exterior domain.

Outline



It is a Schwarz-like method: reformulate the transmission conditions on Γ

$$
\begin{aligned}
(\mathbf{H}_{\;j\Gamma} \quad \mathbf{n}) + \mathbf{T}\;\;(\mathbf{E}_{\;j\Gamma} \quad \mathbf{n}) &= (\mathbf{H}_{+j\Gamma} \quad \mathbf{n}) + \mathbf{T}\;\;(\mathbf{E}_{+j\Gamma} \quad \mathbf{n}) \\
(\mathbf{H}_{+j\Gamma} \quad \mathbf{n}) \quad \mathbf{T}_{+}(\mathbf{E}_{+j\Gamma} \quad \mathbf{n}) &= (\mathbf{H}_{\;j\Gamma} \quad \mathbf{n}) \quad \mathbf{T}_{+}(\mathbf{E}_{\;j\Gamma} \quad \mathbf{n})
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Unknowns of the weak coupling

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Affine resolution operators (the pre-existing solvers)

$\mathbf{R}\ \ \mathbf{g} = \mathbf{E}_{\ j\Gamma}\quad \mathbf{n}$ (`FEM solver`) and $\mathbf{R}_{+}\mathbf{g} = \mathbf{E}_{+j\Gamma}\quad \mathbf{n}$ (`BEM solver`)

where $(\mathbf{E}\ \ ; \mathbf{H}\ \ )$ are the solutions of the following boundary-value problems:

$$\begin{aligned}\mathbf{curl}\ \mathbf{E}\qquad k\ \ Z\ \ \mathbf{H}\qquad &= \mathbf{0};\ \text{in } \Omega\\ \mathbf{curl}\ \mathbf{H}\ \ + k\ \ Z\ \ {}^{1}\mathbf{E}\qquad &= \mathbf{0};\ \text{in } \Omega\\ (\mathbf{H}_{\ j\Gamma}\quad \mathbf{n})\quad \mathbf{T}\ \ (\mathbf{E}_{\ j\Gamma}\quad \mathbf{n})\ &= \mathbf{g};\ \text{on } \Gamma\\ Z_{+}\mathbf{H}_{\mathbf{sc}}\quad \frac{\mathbf{x}}{k\mathbf{x}k}\quad \mathbf{E}_{\mathbf{sc}}\qquad &= \underset{r\ !\ +1}{O}(r\ \ ^{2}).\end{aligned}$$

The Weak FEM-BEM Coupling

! Rewriting of the reformulated transmission conditions

The weak FEM-BEM coupling

$$(\mathbf{Id} \quad S\ ) \begin{matrix} \mathbf{g} \\ \mathbf{g}_+ \end{matrix} \quad = \quad \begin{matrix} \mathbf{0} \\ \mathbf{0} \end{matrix}$$

$$\text{with } S_{\boldsymbol{\pi}} = \begin{matrix} 0 & \mathbf{S}_+ \\ \mathbf{S} & 0 \end{matrix}\ , \mathbf{S} \quad = \mathbf{Id} \quad (\mathbf{T} \quad + \mathbf{T}_+)\mathbf{R}$$

Reformulation of the weak FEM-BEM coupling

The weak FEM-BEM coupling can be recast into a linear system:

$$(\mathbf{Id} \quad A) \begin{matrix} \mathbf{g} \\ \mathbf{g}_+ \end{matrix} = \begin{matrix} \mathbf{b} \\ \mathbf{b}_+ \end{matrix} \quad (\, | \,),$$

where the information about the incident plane wave is contained in the right-hand side. The quantities $\mathbf{g}$ and $\mathbf{g}_+$ can be interpreted as some exchanged information from Ω_+ to Ω and from Ω to Ω_+, respectively.

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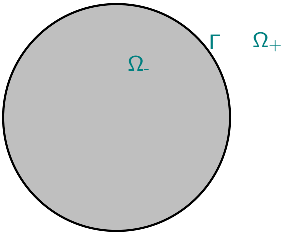
At each iteration of a **Krylov subspace solver** of (|):



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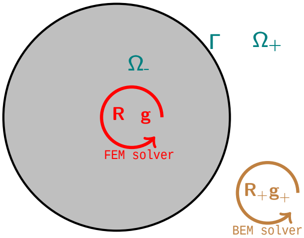

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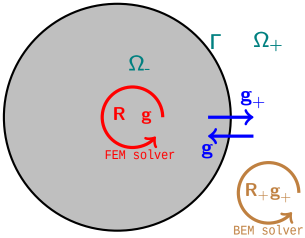
Continuous approach - The weak FEM-BEM coupling formulation

At each iteration of a **Krylov subspace solver** of (|):

1. Solve the interior problem and the exterior problem.
2. Update the interface unknowns $\mathbf{g}_+$ and $\mathbf{g}$ (using A).



Correctly Choosing the Transmission Operators

The weak coupling can **only be solved iteratively**, typically using the GMRES method.
$\rightarrow$ The GMRES convergence of the weak coupling is fundamentally related to the choice of the transmission operators.

Optimal transmission operators for the weak coupling

[Caudron, Geuzaine, Antoine, 2018]

$$\mathbf{T}_- = \boldsymbol{\Lambda}_{+,k_+,Z_+} \Rightarrow \tilde{\mathbf{S}}_+ = \mathbf{0}$$
$$\mathbf{T}_+ = \boldsymbol{\Lambda}_{-,k_-,Z_-} \Rightarrow \tilde{\mathbf{S}}_- = \mathbf{0}$$
$$\Rightarrow Sp(\mathbf{Id} - A) = \{1\}$$

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Remarks: Unlike the interior problem (complex **nonlocal** interactions), the solution of the exterior domain is **spatially localized**.
$\rightarrow$ Use accurate representation of $\mathbf{T}_- = \boldsymbol{\Lambda}_{+,k_+,Z_+}$ and rough computation of $\mathbf{T}_+$ (poor approximation of $\boldsymbol{\Lambda}_{+,k_+,Z_+}$ or $\boldsymbol{\Lambda}_{-,k_-,Z_-}$).

$\Rightarrow$ A nilpotent (leading fast convergence of the GMRES)

Approximating The **MtE** Operators

What about the practical side?

In practice, evaluating **MtE** operators leads to high computational costs. Several practical localized approximations of the **MtE** operators were proposed in literature.

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Low order approximations

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Low order approximations

$$\mathbf{\Lambda}^{0}_{;k\ ;Z} = \frac{1}{Z}(\mathbf{Id}\ \ \mathbf{n})$$

Padé-localized square-root approximations [El Bouajaji, Antoine, Geuzaine, 2014]

$$\mathbf{\Lambda}^{sq;N_p;\ p}_{;k\ ;Z} =$$

$$\frac{1}{Z}\ \ C_0 + \sum_{l=1}^{N_p} A_l\ \ r_\Gamma(\tfrac{1}{k^2}div_\Gamma)\ \ \mathbf{curl}_\Gamma(\tfrac{1}{k^2}curl_\Gamma\ \ \ \mathbf{Id} + B_l\ \ r_\Gamma(\tfrac{1}{k^2}div_\Gamma)\ \ \mathbf{rot}_\Gamma(\tfrac{1}{k^2}rot_\Gamma\ \ {}^{1}\ !\ {}^{1}$$

$$\mathbf{Id}\ \ \mathbf{curl}_\Gamma(\tfrac{1}{k^2}curl_\Gamma)\ \ (\mathbf{Id}\ \ \mathbf{n})$$

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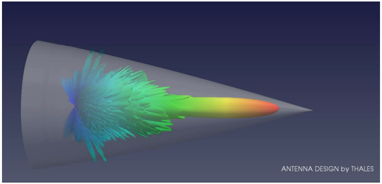


FEM solver

`GmshFEM`,
a newly developed open-source finite element library based on `Gmsh`

BEM solver

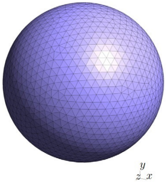
`Bempp-cl`,
an open-source boundary element library

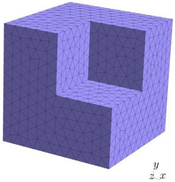
The transmission operators selected

$$(\mathbf{T} \quad ; \mathbf{T}_+)$$

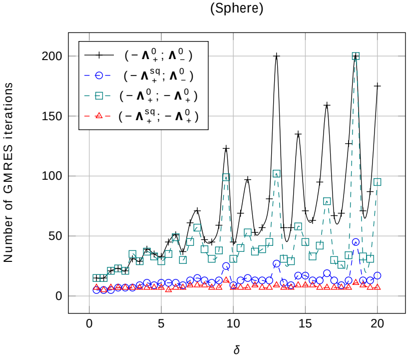
We suppose

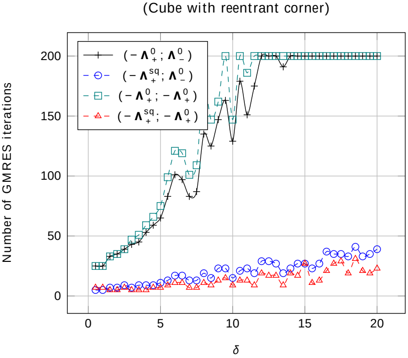
$$k \; = \; k_+ e^{\quad kxk^2} \text{ and } Z \; = \frac{Z_+}{} e^{kxk^2} \text{ with } \quad 2\, \mathbb{R}_+ \text{ and } Z_+ = Z_0$$



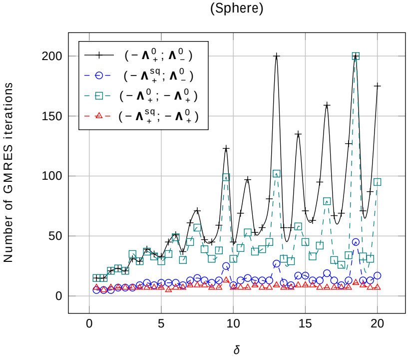


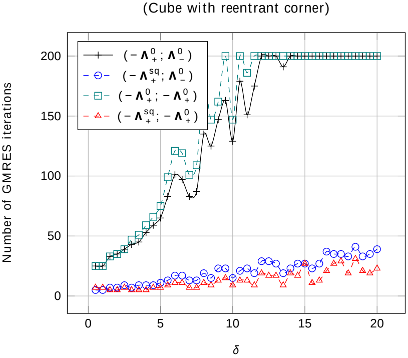
Number Of GMRES Iterations *Vs.*





5 points per wavelength and $k_+ = 1$





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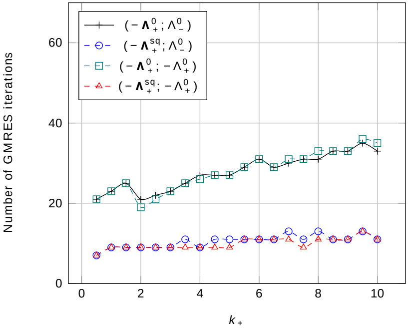
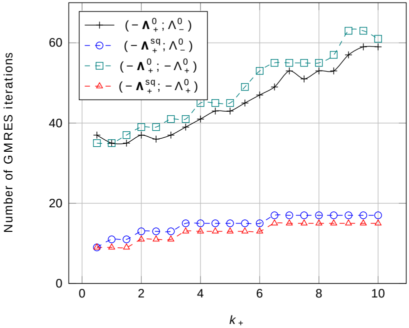
! **Superior iterative performance** when $\mathbf{T}_+ = \quad \boldsymbol{\Lambda}^0_{+;k_+;Z_+}$

Number Of GMRES Iterations *Vs.* k_+

(Sphere)

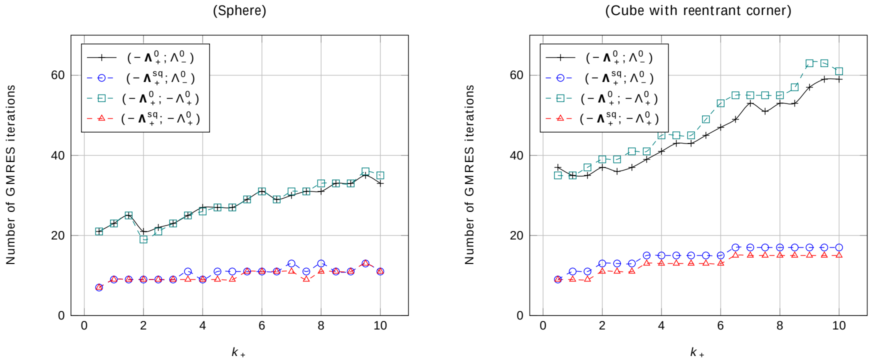


(Cube with reentrant corner)



5 points per wavelength, $= 2$

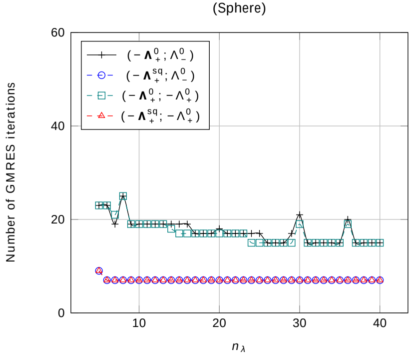
Number Of GMRES Iterations *Vs.* k_+

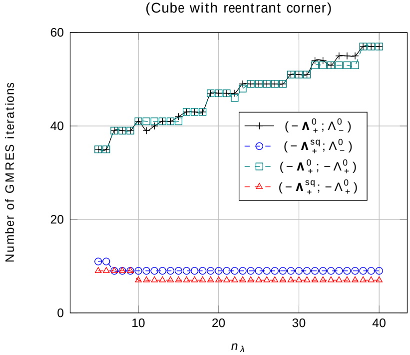


5 points per wavelength, $\quad = 2$

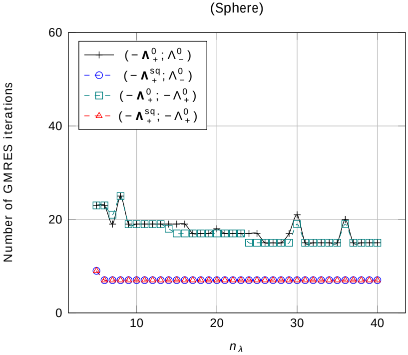
! $(\quad \boldsymbol{\Lambda}^{sq;N_p;\ p}_{+;k_+;Z_+}; \quad \boldsymbol{\Lambda}^0_{+;k_+;Z_+})$ leads to the **lowest dependency** of the GMRES iterations with respect to k_+

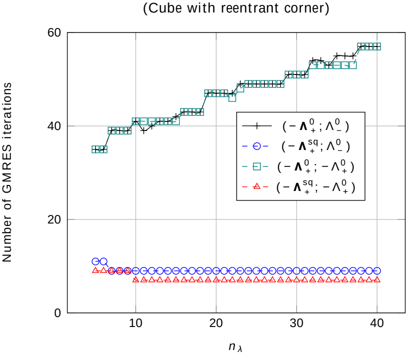
Influence Of Mesh Refinement





$= 2$ and $k_+ = 1$





$= 2$ and $k_+ = 1$

! Convergence clearly **deteriorates** with the pairs ($\boldsymbol{\Lambda}^0_{+;k_+;Z_+}$; $\boldsymbol{\Lambda}^0_{;k\ ;Z}$)

Relative differences in $\mathbf{L}^2_t(\Gamma)$-norm between the strong and the weak FEM-BEM coupling electric/magnetic currents

$$e_{\mathbf{H}\quad ;h_{j\Gamma}}\quad \mathbf{n} = \frac{k\mathbf{H}\quad ;h_{j\Gamma}\quad \mathbf{n}\quad \mathbf{H}^{\mathrm{Ref}}_{j\Gamma}\quad \mathbf{n}k_{\mathbf{L}^2_t(\Gamma_h)}}{k\mathbf{H}^{\mathrm{Ref}}_{j\Gamma}\quad \mathbf{n}k_{\mathbf{L}^2_t(\Gamma_h)}};$$

$$e_{\mathbf{E}\quad ;h_{j\Gamma}}\quad \mathbf{n} = \frac{k\mathbf{E}\quad ;h_{j\Gamma}\quad \mathbf{n}\quad \mathbf{E}^{\mathrm{Ref}}_{j\Gamma}\quad \mathbf{n}k_{\mathbf{L}^2_t(\Gamma_h)}}{k\mathbf{E}^{\mathrm{Ref}}_{j\Gamma}\quad \mathbf{n}k_{\mathbf{L}^2_t(\Gamma_h)}}:$$

Accuracy of the weak FEM-BEM coupling

Relative differences in $\mathbf{L}_t^2(\Gamma)$-norm between the strong and the weak FEM-BEM coupling electric/magnetic currents

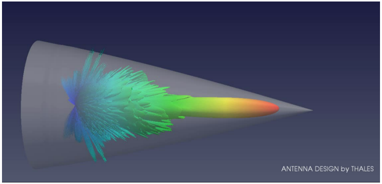
$$e_{\mathbf{H}\ ;h_{j\Gamma}\ \mathbf{n}} = \frac{k\mathbf{H}\ ;h_{j\Gamma}\ \ \mathbf{n}\ \ \mathbf{H}^{\text{Ref}}_{j\Gamma}\ \ \mathbf{n}k_{\mathbf{L}_t^2(\Gamma_h)}}{k\mathbf{H}^{\text{Ref}}_{j\Gamma}\ \ \mathbf{n}k_{\mathbf{L}_t^2(\Gamma_h)}};$$

$$e_{\mathbf{E}\ ;h_{j\Gamma}\ \mathbf{n}} = \frac{k\mathbf{E}\ ;h_{j\Gamma}\ \ \mathbf{n}\ \ \mathbf{E}^{\text{Ref}}_{j\Gamma}\ \ \mathbf{n}k_{\mathbf{L}_t^2(\Gamma_h)}}{k\mathbf{E}^{\text{Ref}}_{j\Gamma}\ \ \mathbf{n}k_{\mathbf{L}_t^2(\Gamma_h)}}:$$

k_+		$e_{\mathbf{E}\ ;h_{j\Gamma}\ \mathbf{n}}$	$e_{\mathbf{H}\ ;h_{j\Gamma}\ \mathbf{n}}$
1	2	9.9%	0.8%
2	2	7.5%	0.8%
1	3	5.7%	4.9%
1	4	2.9%	1.0%

$e_{\mathbf{E}\ ;h_{j\Gamma}\ \mathbf{n}}$ and $e_{\mathbf{H}\ ;h_{j\Gamma}\ \mathbf{n}}$ for the sphere and for di erent k_+ and $\ \ (n\ \ = 10)$

Outline



FEM solver

`GmshFEM`

BEM solver

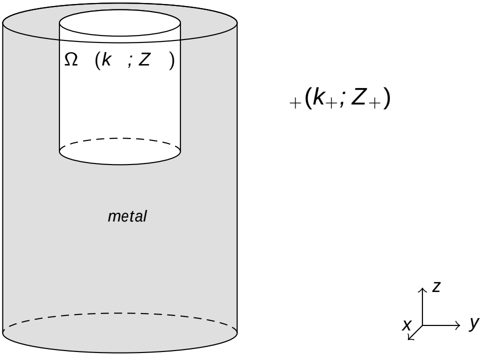
`Antenna Design` (AD) coupled with the H-Matrix library `Hi-BoX`, the Thales internally developed code

Validation

The test cases were validated using AD coupled with `Hi-BoX`

The transmission operators selected

$$(\mathbf{T}\ ; \mathbf{T}_{+}) = (\ \ \mathbf{\Lambda}_{+}^{\mathrm{sq}};\ \ \mathbf{\Lambda}_{+}^{0})$$



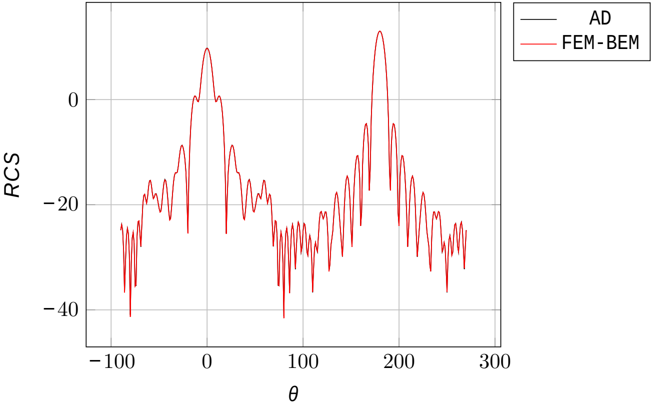
Incident plane wave linearly polarized along z

Frequency: $10[GHz]$

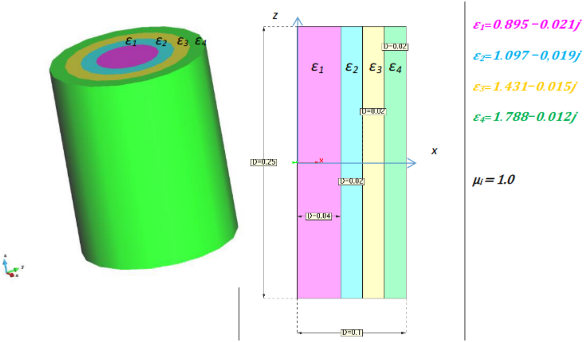
$= (0{:}895 \quad 0{:}021)\ _0$, $= \ _0$,

$_+ = \ _0$, $_+ = \ _0$

The bistatic RCS in plane $y = 0$



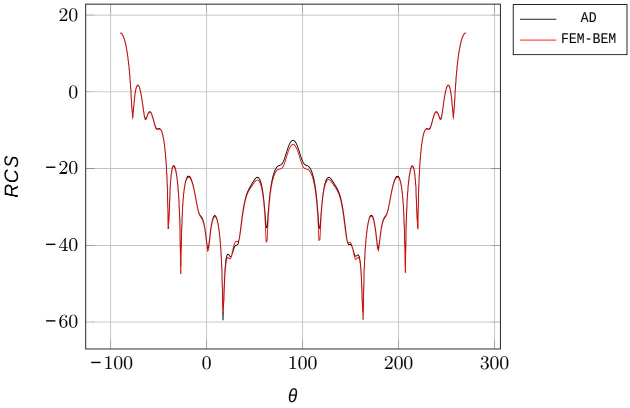
Good convergence of GMRES in 11 iterations



Incident plane wave linearly polarized along x

Frequency: $5[GHz]$

The bistatic RCS in plane $y = 0$



Good convergence of GMRES in 15 iterations

A K/KA Dual Band Polarizer

We compute the RHCP gain and the LHCP gain in the *yz* plane defined by

$$RHCP(\ ;90) = 20\log(|\frac{1}{\sqrt{2}}(E^{1}_{sc;}\ (\ ;90) + \ E^{1}_{sc;}\ (\ ;90)|);$$

$$LHCP(\ ;90) = 20\log(|\frac{1}{\sqrt{2}}(E^{1}_{sc;}\ (\ ;90) \quad \ E^{1}_{sc;}\ (\ ;90)|);$$

where we denote the components of the far-field by $E^{1}_{sc;}$ and $E^{1}_{sc;}$

A K/KA Dual Band Polarizer

A K/KA Dual Band Polarizer

The polarizer is tested with two linearly polarized rectangular horns, one for each band ($20.2[GHz]$ and $30[GHz]$)

This problem can be challenging due to the rather small thickness and intricacies of the polarizer

For more information: P. Naseri et al, *Dual-Band Dual-Linear-to-Circular Polarization Converter in Transmission Mode Application to K/Ka -Band Satellite Communications*, in IEEE, 2018.

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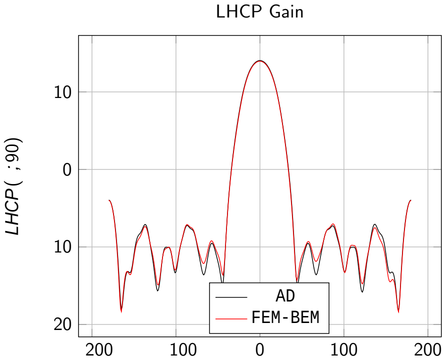
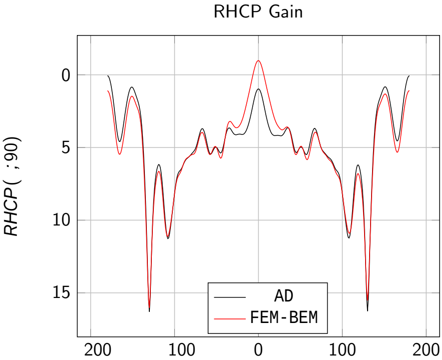
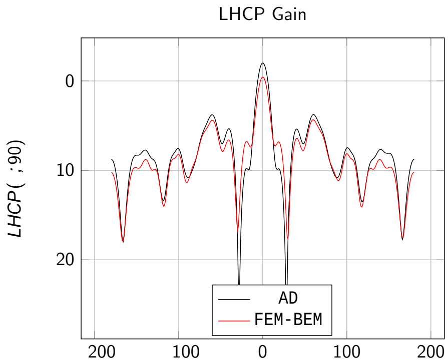
30[GHz]

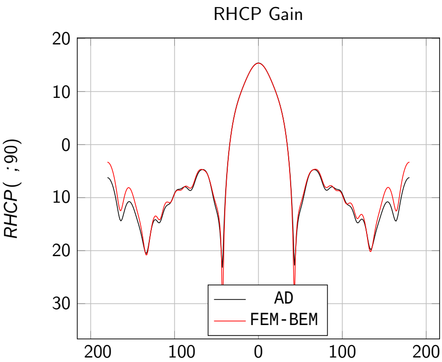




Good convergence of GMRES in 15 iterations

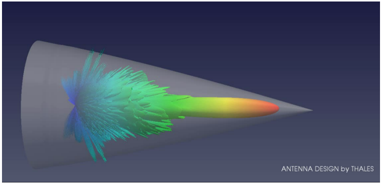
20:2[GHz]





Good convergence of GMRES in 17 iterations

Outline



Conclusion

Features of the new weak FEM-BEM coupling

- The convergence rate is **slightly dependent** on the geometry configuration, the frequency k, the mesh refinement and the contrast between the two subdomains.
- Allows to reuse optimized pre-existing solvers.
- For a relevant accuracy on the far-field, the number of iterations is very small and stable (typically about 15).

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Perspectives

DDM for FEM part.

Perspectives

DDM for FEM part.

(a) Concentric partition

(b) Partition with corners

	Concentric partition	Partition with junctions
GMRES iterations	9	30
$e_{\mathbf{E}\quad ;h_{j\Gamma}}\quad \mathbf{n}$	1.8%	3.8%
$e_{\mathbf{H}\quad ;h_{j\Gamma}}\quad \mathbf{n}$	0.5%	2.6%

I. Badia, C. Caudron, X. Antoine and C. Geuzaine. *A well-conditioned weak coupling of boundary element and high-order finite element methods for time-harmonic electromagnetic scattering by inhomogeneous objects*, to be published in SIAM Journal on Scientific Computing, 2022

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Thank you for your attention !

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